Logics of agency

Chapter 3: The modal view of agency

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OVERVIEW OF THIS CHAPTER

- Generalities and justifications
- Seeing to it that
  - Various modalities and original semantics
- Bringing it about
  - Several semantics

OUTLINE

1. GROUNDING AGENCY AS A MODALITY: A LINGUISTIC AGENDA

2. THE FAMILY OF SEEING TO IT THAT

3. BRINGING IT ABOUT

4. ADVANCED: AXIOMATIZATION OF THE INDIVIDUAL CHELLAS' STIT

Von Wright can be considered a pioneer in modern philosophy of action.

> *It would not be right, I think to call acts a kind or species of events. An act is not a change in the world. But many acts may quite appropriately be described as the bringing about or effecting ('at will') of a change. To act is, in a sense, to interfere with 'the course of nature'. ([von Wright 63])*

Agency is seen as a modal notion instead of a mere referent in the language.

Lacking ontological structure and handwaving for some; source of real mathematical structure for others.

In 1991, Belnap writes:

> *The modal logic of agency is not popular. Perhaps largely due to the influence of Davidson (see the essays in Davidson 1980), but based also on the very different work of such as Goldman 1970 and Thomson 1977, the dominant logical template takes an agent as a wart on the skin of an action, and takes an action as a kind of event. This "actions as events" picture is all ontology, not modality, and indeed, in the case of Davidson, is driven by the sort of commitment to first-order logic that counts modalities as Bad.* *[Belnap 1991]*

St. Anselm (11th century): If *a* does something he does so such that something is true or false. ([Henry 1953], [Chisholm 1964])

- The relevant aspect of agency is what actions bring about.
- No matter how the structure of the action.

The King is responsible for Anselm being in exile

$\Leftrightarrow$

The King sees to it that Anselm is in exile

$\triangledown_{King}$"Anselm is in exile"

Grounding the modality of agency: Belnap and Perloff's linguistic agenda

Problem definition: distinguish between sentences which involve agency and those which do not.

- Is "Queequeg struck home with his harpoon" agentive for Queequeg?
  - try to uncover general principles for deciding whether a sentence is agentive
- An agentive sentence must emphasize a sort of causality and responsability of an agent for the truth of a state of affairs.

Fundamental theses [Belnap and Perloff 1988]

Definition (Paraphrase thesis)

The sentence φ marks the agentiveness of agent *a* just in case φ may be usefully paraphrased as "*a* sees to it that φ".

Definition (Agentiveness thesis)

The sentence "*a* sees to it that φ" is agentive for *a*.

Von Wright, Chisholm, Kenny, Castañeda: some sentences should be excluded from being the complement. Belnap and Perloff consider this unnecessary.

Definition (Complement thesis)

The sentence "*a* sees to it that φ" is grammatical and meaningful for any sentence φ.

IS QUEEQUEG AGENTIVE?

sentence	careful English
Queequeg struck home with his harpoon.	agentive
Queequeg's harpoon struck home.	non agentive

But these two sentences are equivalently agentive:

- Queequeg sees to it that Queequeg struck home with his harpoon.
- Queequeg sees to it that Queequeg's harpoon struck home.

(We use $\triangledown_a \varphi$ as a generic notation for "agent *a* does φ".)

A long history of argument and disagreement [Chellas 69], [Pörn], [Jones & Pörn], [Elgesem 93]:

(M)	$\triangledown_a(\varphi \wedge \psi) \rightarrow (\triangledown_a \varphi \wedge \triangledown_a \psi)$
(C)	$(\triangledown_a \varphi \wedge \triangledown_a \psi) \rightarrow \triangledown_a(\varphi \wedge \psi)$
(N)	$\triangledown_a \top$
(No)	$\neg \triangledown_a \top$
(T)	$\triangledown_a \varphi \rightarrow \varphi$
(RE)	if $\varphi \leftrightarrow \psi$ then $\triangledown_a \varphi \leftrightarrow \triangledown_a \psi$

Here, agency does not require intention!

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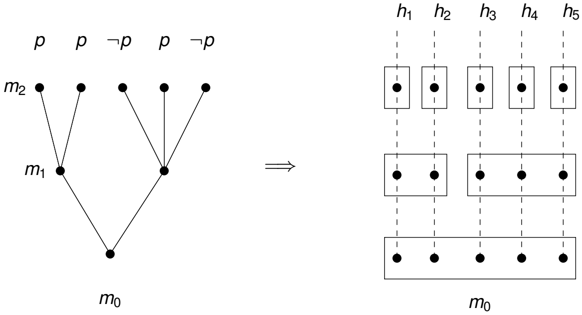
Several logics

- Achievement stit [Belnap and Perloff 1988]
- Deliberative stit [von Kutschera 1986], Chellas stit [Horty and Belnap 1995]
- “Operator of Chellas” [Chellas 1969]
- Strategic stit [Horty 2001], [Belnap et al. 2001], ...
- ...

OCKHAMIST BRANCHING TIME TEMPORAL LOGIC (BT)

BT structure $\langle Mom, < \rangle$:



- History = maximally $<$-ordered set of moments
- $Hist$ = set of all histories
- H_m = set of histories passing through the moment m
- Explode moments into indexes (moment/history pairs)
  - $m_0/h_3 \not\models \mathbf{F}p$
  - $m_0/h_1 \models \mathbf{F}p$

Agents and Choices (AC): selecting sets of histories

The notions of a history and history contingency are central to capture the essence of agency. Belnap et al. illustrate it with the following quote:

> *When Jones butters the toast [...] the nature of his act, on this view, is to constrain the history to be realized so that it must lie among those in which he butters the toast. Of course, such an act still leaves room for a good deal of variation in the future course of events, and so cannot determine a unique history; but it does rule out all those histories in which he does not butter the toast. ([Belnap et al. 2001])*

Choice

- $Choice : \text{Agt} \times Mom \to \mathcal{P}(\mathcal{P}(Hist))$
  - $Choice(a, m)$ = repertoire of choices for agent a at moment m
  - $Choice$ is a function mapping each agent and each moment m into a partition of H_m
- $Choice(a, m) : Hist \to \mathcal{P}(Hist)$
  - For $h \in H_m$: $Choice(a, m)(h)$ = the particular choice of a at index m/h.
- Independence of agents/choices: Let h, m.
  For all collections of $X_a \in Choice(a, m)(h)$, $\bigcap_{a \in Agt} X_a \neq \emptyset$.
- No choice between undivided histories: if $\exists m' > m$ s.t. $h, h' \in H_{m'}$ then $h' \in Choice(a, m)(h)$.

Choice OF GROUPS

A coalition (or group) is a set $C \subseteq \mathrm{Agt}$.

We define:

$$Choice(C, m)(h) = \bigcap_{a \in C} Choice(a, m)(h)$$

(Already due to [von Kutschera 1986].)

A $BT + AC$ model is a tuple $\mathcal{M} = \langle Mom, <, Choice, v \rangle$, where:

- $\langle Mom, < \rangle$ is a BT structure;
- $Choice : \mathrm{Agt} \times Mom \to \mathcal{P}(\mathcal{P}(Hist))$;
- v is valuation function $v : \mathrm{Prop} \to \mathcal{P}(Mom \times Hist)$.

Example: Going Aboard

In Chapter 21 of Melville's Moby Dick, Queequeg and Ishmael go aboard the Pequod deliberately and Ishmael is well. Ishmael sees to it that he is on board the Pequod. This is the real history.

In some alternative histories:

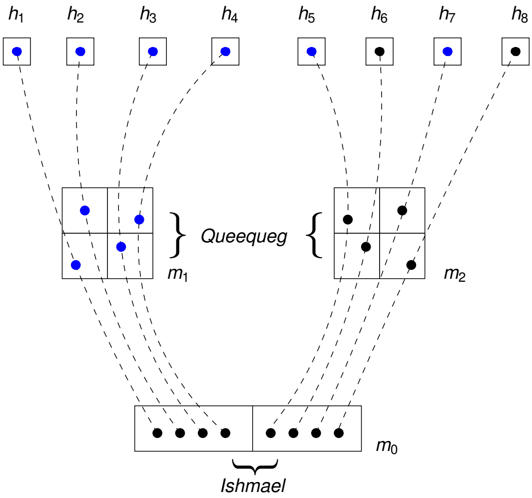
- Ishmael could have stayed on the docks and walked away
- Queequeg could have knocked him unconscious and drag him to the engine room aboard

How agentive are the characters for Ishmael sailing on board the Pequod?

On the next slide:

- m_0 Ishmael can go aboard, or stay on the docks
- m_1 Ishmael can stay on the deck or walk to the engine room
- m_1 Queequeg can do nothing or, knock Ishmael out and drag him to the engine room
- m_2 Ishmael can stay by or, walk away
- m_2 Queequeg can do nothing or, knock Ishmael out and drag him on board to the engine room

CORRESPONDING BT+AC MODEL



WE PRESENT SEVERAL MODALITIES OF AGENCY

$BT + AC$ models are enough for:

- Deliberative stit
- Horty's "Chellas" stit
- Strategic Chellas stit
- Strategic Chellas stit of ability

$BT + AC$ models are not enough for:

- Achievement stit
- Operator of Chellas
- Strategic achievement stit

A $BT + AC + I$ model is a tuple
$\mathcal{M} = \langle Mom, <, Choice, v, Instant \rangle$ where:

- $\langle Mom, <, Choice, v, \rangle$ is a $BT + AC$ model
- $Instant$ is a partition of Mom
  - Unique intersection: if $I \in Instant$ and $h \in Hist$ then $I \cap h$ is a singleton $\{m_{I,h}\}$
  - Order preservation: if $m_{I_1,h_1} < m_{I_2,h_1}$ then $m_{I_1,h_2} < m_{I_2,h_2}$

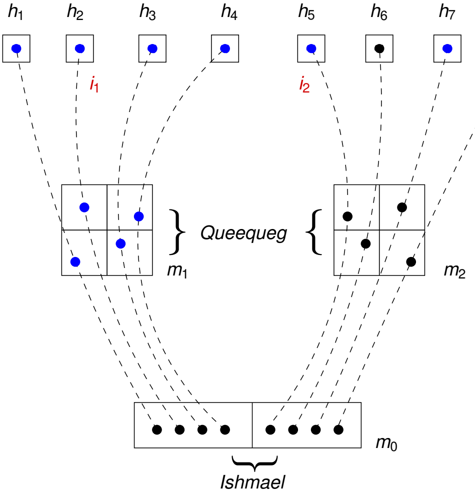
We note $I(m)$ the partition of $Instant$ containing moment m.

ACHIEVEMENT STIT [BELNAP AND PERLOFF 1988]

An agent *i* sees to it that φ if a prior choice of *i* made sure that φ is true at the current instant, and φ could have been false at this instant had *i* done otherwise.

$M, m/h \models [a\, astit : \varphi]$ iff

there is a witness moment $m_0 < m$ such that
(+) $M, m'/h' \models \varphi$ for every h' and m' such that
(i) $Choice(a, m_0)(h) = Choice(a, m_0)(h')$;
(ii) $m' \in h'$ and $I(m) = I(m')$;
(–) $M, m''/h'' \not\models \varphi$ for some m'' and h''
such that $I(m'') = I(m)$ and $h'' \in m''$



$i_1 \models [\textit{Ishmael astit} : \bullet]$

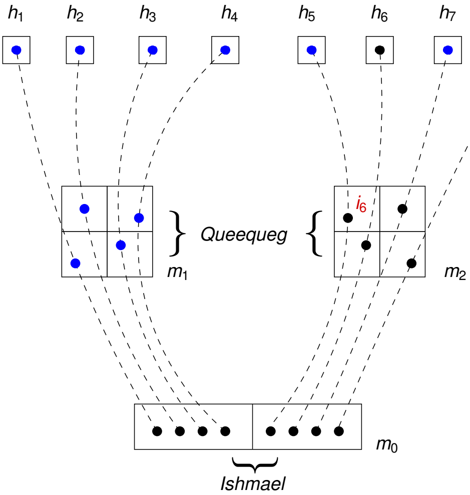
$i_2 \models \bullet \land \neg[\textit{Ishmael astit} : \bullet]$

DELIBERATIVE STIT [VON KUTSCHERA 1986]

The witness moment is the current moment: *a* currently chooses φ but φ was not inevitable.

$M, m/h \models [a\, dstit : \varphi]$ iff

> ***(+)*** $M, m/h' \models \varphi$ for all $h' \in Choice(a, m)(h)$
> ***(–)*** $M, m/h'' \models \neg\varphi$ for some $h' \in H_m$

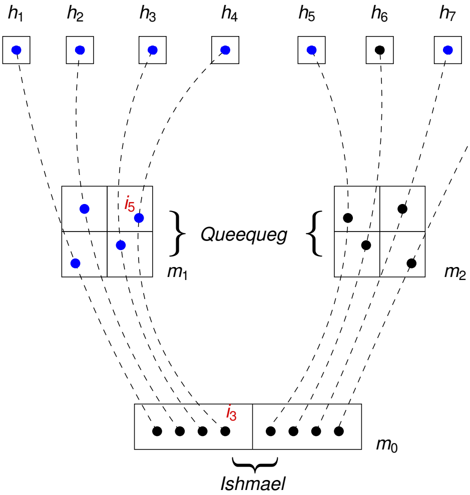


$i_6 \models [Queequeg\ dstit : X\bullet]$

"CHELLAS" STIT [HORTY AND BELNAP 1995]

Like the deliberative stit, but without the negative condition.

$M, m/h \models [a\,cstit\colon \varphi]$ iff $M, m/h' \models \varphi$ for all $h' \in \mathit{Choice}(a, m)(h)$



$i_3 \models [\mathit{Ishmael\ cstit} : X\bullet]$

$i_5 \models [\mathit{Ishmael\ cstit} : \bullet]$

$i_5 \models \neg[\mathit{Ishmael\ dstit} : \bullet]$

"CHELLLAS" STIT TO CAPTURE CHOICES IN MATRIX GAMES

Consider the moment m_{game}:

	defect_b	silent_b
defect_a	$h_{(-6,-6)}$	$h_{(-10,0)}$
silent_a	$h_{(0,-10)}$	$h_{(-2,-2)}$

At $m_{game}/h_{(-6,-6)}$, agent a sees to it that $h_{(-6,-6)} \vee h_{(-10,0)}$.
At $m_{game}/h_{(-2,-2)}$, the coalition $\{a, b\}$ see to it that $h_{(-2,-2)}$.

The operator of Chellas [Chellas 1969]

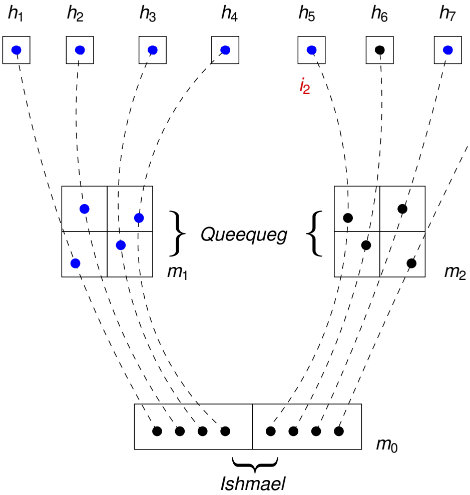
The semantics of the operator of Chellas (Δ_a) requires a discrete time.

$M, m/h \models \Delta_a \varphi$ iff

(let m_{-1} the moment immediately preceding m)
(+) $M, m'/h' \models \varphi$ for every h' and m' such that
(i) $I(m) = I(m')$ and $m' \in h'$;
(ii) $Choice(a, m_{-1})(h) = Choice(a, m_{-1})(h')$.

Horty's "Chellas" stit $\neq$ the operator of Chellas

Δ ON OUR MODEL

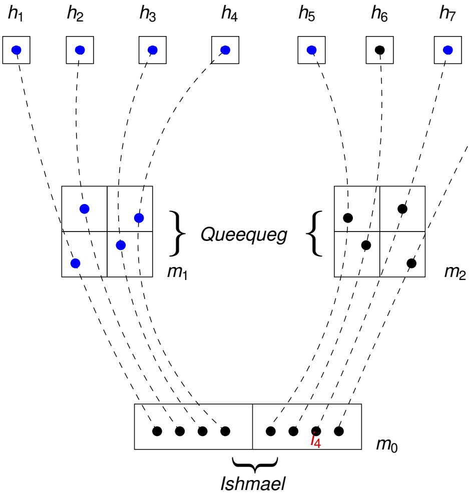


$i_2 \models \Delta_{Queequeg}$

$i_2 \models \neg\Delta_{Ishmael}$

Strategic stit (informally and with pointers)

- Belnap & Perloff's "strategic achievement stit"
  - [Belnap et al 2001, Ch. 13]
  - "There have been a series of choices by agent *a* in the past that ensured φ at the current index."
- Horty's "strategic Chellas stit"
  - [Horty 2001, Ch. 7]
  - [Broersen et al. 2006 JELIA] [Troquard & Walther 2012]
  - "The current series of futures choices (strategy) by agent *a* ensure φ to be realised."
- Horty's "strategic Chellas stit of ability"
  - [Horty 2001, Ch. 7]
  - Relationship with ATL [Alur et al. 2002] [Broersen et al. 2006 JLC]
  - "There is a series of future choices by agent *a* that (would) ensure φ to be realised."



$i_4, \{m_1 \mapsto \{h_5, \ldots, h_8\}, m_2 \mapsto \{h_7\}\} \models$
$[\{\mathit{Ishmael}, \mathit{Queequeg}\}\ \mathit{scstit}: XX\bullet]$

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2 The family of Seeing To It That

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Semantics combining normal modalities [Pörn 1970]

- $D_a\varphi$ is true in a world w if φ is true at every hypothetical situation where agent a "does at least as much as he does in w"
- $D'_a\varphi$ is true in w if $\neg\varphi$ is true in every hypothetical situation w' such that "the opposite of everything a does in w is the case in w'"
- Combination of two normal operators in a non-normal modality:
  - $D_a\varphi$: "it is necessary for something a does that φ"
  - $D'_a\varphi$: "but for a's action, it would not be the case that φ"
  - $E_a\varphi \triangleq D_a\varphi \wedge \neg D'_a\neg\varphi$ reads "agent a brings it about that φ".

- “one problem with the proposed semantics is that ‘doing at least as much as’ he does in [a world], and the notion of an agent doing ‘the opposite’ of everything he does in [a world], are of dubious intelligibility without substantial further elucidation, and Pörn offers none.” [Horgan 1979]
- “the intuitive significance of this semantics is not altogether clear.” [Segerberg 1992]

SELECTION FUNCTIONS $M = \langle W, \{f_i\}, V \rangle$

Inspired by [Sommerhoff 69] control theory.

- W is some set of possible worlds,
- V : Prop $\longrightarrow \mathcal{P}(W)$ is a valuation function
- $f_i : W \times \mathcal{P}(W) \to \mathcal{P}(W)$ is a selection function for every agent i

The object $f_i(w, X)$ is the set of those worlds where i realizes the ability he has in w to bring about his goal X;

i is able to bring about X at w if $f_i(w, X)$ is nonempty;

i brings about X at w if w belongs to $f_i(w, X)$.

Selection functions $M = \langle W, \{f_i\}, V \rangle$ (ctd)

The functions f_i have to satisfy the following constraints:

- $f_i(w, X) \subseteq X$, for every $X \subseteq W$ and $w \in W$;
- $f_i(w, X_1) \cap f_i(w, X_2) \subseteq f_i(w, X_1 \cap X_2)$, for every $X_1, X_2 \subseteq W$ and $w \in W$;
- $f_i(w, W) = \emptyset$, for every $w \in W$.

The truth conditions are as follows:

$$\begin{array}{lll} M, w \models p & \text{iff} & w \in V(p); \\ M, w \models E_i\varphi & \text{iff} & w \in f_i(w, ||\varphi||^M); \\ M, w \models C_i\varphi & \text{iff} & f_i(w, ||\varphi||^M) \neq \emptyset. \end{array}$$

In the last two conditions the set $||\varphi||^M$ is the extension of φ in M, i.e. the set of possible worlds where φ is true: $||\varphi||^M = \{w \in W \mid M, w \models \varphi\}$.

Paraphrased from [McNamara 2000]:

- The semantics involves some agents Agt, existing at various possible worlds W.
- In these worlds, an agent often exhibits her agency by bringing certain things about.
  - $EE : W \times \text{Agt} \longrightarrow \mathcal{P}(\mathcal{P}(W))$
- Presumably, she does so by taking certain actions that result in certain propositions being true, the ones she has brought about.

Neighbourhoods semantics $M = \langle W, EE, EC, V \rangle$

- W is some set of possible worlds,
- $V : \text{Prop} \longrightarrow \mathcal{P}(W)$ is a valuation function
- $EE : W \times \text{Agt} \longrightarrow \mathcal{P}(\mathcal{P}(W))$
- $EC : W \times \text{Agt} \longrightarrow \mathcal{P}(\mathcal{P}(W))$

Constraints on the neighborhoud functions:

- $W \notin EE(w, i)$
- $\emptyset \notin EC(w, i)$
- if $X \in EE(w, i)$, then $w \in X$
- if $X \in EE(w, i)$ and $Y \in EE(w, i)$ then $X \cap Y \in EE(w, i)$
- $EE(w, i) \subseteq EC(w, i)$

Truth values:

$$
\begin{array}{lll}
M, w \models p & \text{iff} & w \in V(p); \\
M, w \models E_i\varphi & \text{iff} & ||\varphi||^M \in EE(w, i); \\
M, w \models C_i\varphi & \text{iff} & ||\varphi||^M \in EC(w, i).
\end{array}
$$

[Elgesem 93], [Elgesem 97], [Governatori & Rotolo 2005], ...:

- Propositional logic
- $\vdash \neg E_i \top$
- $\vdash E_i\varphi \wedge E_i\psi \to E_i(\varphi \wedge \psi)$
- $\vdash E_i\varphi \to \varphi$
- if $\vdash \varphi \leftrightarrow \psi$ then $\vdash E_i\varphi \leftrightarrow E_i\psi$

ELGESEM'S "BRINGING-IT-ABOUT" AND ABILITY [ELGESEM 93]

$$E_i\varphi \rightarrow C_i\varphi$$

Also, for coalitions [Troquard 2014]:

$$E_{G_1}\varphi \wedge E_{G_2}\varphi \rightarrow C_{G_1 \cup G_2}(\varphi \wedge \psi)$$

Elgesem studied agency and ability through a net of derived concepts.

$$\begin{array}{lcl}
\mathit{Does}_G\varphi & \triangleq & E_G\varphi \\
\mathit{Ability}_G\varphi & \triangleq & C_G\varphi \\
\mathit{Compatible}_G\varphi & \triangleq & \neg E_G\neg\varphi \\
\mathit{Unpreventable}_G\varphi & \triangleq & \neg C_G\neg\varphi \\
\mathit{Independently}_G\varphi & \triangleq & \neg E_G\varphi \wedge \varphi \\
\mathit{Opportunity}_G\varphi & \triangleq & E_G\neg\varphi \vee \varphi
\end{array}$$

Decision problem 1: Is φ a valid formula?
Decision problem 2: Is φ a satisfiable formula?

THEOREM ([TROQUARD 14])

Reasoning about

$$\begin{cases} \textit{individual agency} \\ \textit{individual agency and ability} \\ \textit{coalitional agency and ability} \end{cases}$$

can be solved in space polynomial in the size of φ (PSPACE-easy).

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Historic necessity:

$$m/h \models \Box\varphi \text{ iff } \forall h' \in H_m : m/h' \models \varphi$$

- $[a\, dstit : \varphi] \leftrightarrow [a\, cstit : \varphi] \wedge \neg\Box\varphi$
- $[a\, cstit : \varphi] \leftrightarrow [a\, dstit : \varphi] \vee \Box\varphi$
- $\Box\varphi \leftrightarrow$ ... (see a few slides ahead)

$S5$ MODAL LOGIC [LEWIS, LANGFORD 1932]

- $S5$ is characterized by equivalence frames (reflexive, transitive, and symmetrical).
- Axiomatics: (K, T, 4, B), (K, D, T, 4, 5)...

K $\blacksquare(\varphi \to \psi) \to (\blacksquare\varphi \to \blacksquare\psi)$
T $\blacksquare\varphi \to \varphi$
4 $\blacksquare\varphi \to \blacksquare\blacksquare\varphi$
5 $\blacklozenge\varphi \to \blacksquare\blacklozenge\varphi$
B $\varphi \to \blacksquare\blacklozenge\varphi$
D $\blacksquare\varphi \to \blacklozenge\varphi$

LEMMA

$A_1 A_2 \ldots A_k \varphi \leftrightarrow A_k \varphi$, $A_i \in \{\blacksquare, \blacklozenge\}$.

XU'S *Ldm* AXIOMATICS OF *individual* CHELLAS STIT

Convenient notation:

- $[i]\varphi$ instead of $[\{i\}\ cstit : \varphi]$

S5($\Box$)	axiom schemas of S5 for $\Box$
S5($[i]$)	axiom schemas of S5 for every $[i]$
($\Box \to [i]$)	$\Box\varphi \to [i]\varphi$
(AIA_k)	$(\Diamond[0]\varphi_0 \wedge \ldots \wedge \Diamond[k]\varphi_k) \to \Diamond([0]\varphi_0 \wedge \ldots \wedge [k]\varphi_k)$

THEOREM ([XU 1994])

Ldm is sound and complete w.r.t. BT + AC models.

Clearly via the semantics and the completeness theorem:

$$\vdash [1][0]\varphi \to \Box\varphi$$

- Advanced (?) problem: derive it from *Ldm*. I do not know the solution.

The other way round holds too!

- Simple exercise: derive it from *Ldm*.

Then

- $\vdash \Box\varphi \leftrightarrow [1][0]\varphi$
- we can get rid off the $\Box$ operator!

ALTERNATIVE *Ldm*

- Independence of agents in *Ldm*: (AIA$_k$)
  $\Diamond[0]\varphi_0 \wedge \ldots \Diamond[k]\varphi_k \to \Diamond([0]\varphi_0 \ldots [k]\varphi_k)$
- Alternative axiomatization of *Ldm*
  [Balbiani, Herzig, Troquard 2008]:

S5(i)	enough S5-theorems, for every $[i]$
Def($\Box$)	$\Box\varphi \leftrightarrow [1][0]\varphi$
(GPerm$_k$)	$\langle l\rangle\langle m\rangle\varphi \to \langle n\rangle \bigwedge_{i\in \mathsf{Agt}\setminus\{n\}}\langle i\rangle\varphi$

- (GPerm$_k$) captures independence of agents

All axiom schemes are in the Sahlqvist class, and therefore have a standard possible worlds semantics.

Kripke models are of the form $M = \langle W, R, V \rangle$, where

- W is a nonempty set of possible worlds;
- R is a mapping associating to every $i \in \text{Agt}$ an equivalence relation R_i on W;
- V is a mapping from Prop to the set of subsets of W.

We impose that R satisfies the general permutation property.

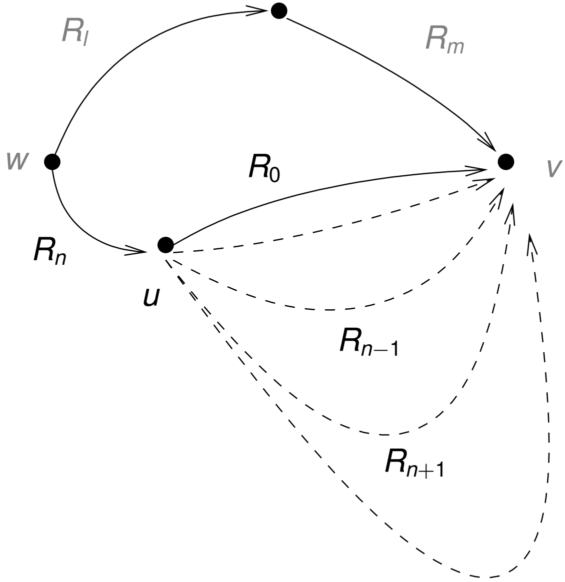
DEFINITION (GENERAL PERMUTATION PROPERTY)

R satisfies the *general permutation property* iff:
for all $w, v \in W$ and for all $l, m, n \in \text{Agt}$, if $\langle w, v \rangle \in R_l \circ R_m$ then there is $u \in W$ such that: $\langle w, u \rangle \in R_n$ and $\langle u, v \rangle \in R_i$ for every $i \in \text{Agt} \setminus \{n\}$.

We have the usual truth condition:

$$M, w \models [i]\varphi \text{ iff } M, u \models \varphi \text{ for every } u \text{ such that } \langle w, u \rangle \in R_i$$

Alternative semantics (illustration)



If $\mathrm{Agt} = \{0, 1\}$ then the validities are axiomatized by:

- Def($\Box$): $\Box\varphi \leftrightarrow [1][0]\varphi$
- S5(0)
- S5(1)
- (GPerm$_1$), two instances:
  - $\langle 1\rangle\langle 0\rangle\varphi \rightarrow \langle 0\rangle\langle 1\rangle\varphi$
  - $\langle 0\rangle\langle 1\rangle\varphi \rightarrow \langle 1\rangle\langle 0\rangle\varphi$

Moreover,

- the permutation axiom $\langle 1\rangle\langle 0\rangle\varphi \leftrightarrow \langle 0\rangle\langle 1\rangle\varphi$
- Church-Rosser axioms $\langle 0\rangle[1]\varphi \rightarrow [1]\langle 0\rangle\varphi$, $\langle 1\rangle[0]\varphi \rightarrow [0]\langle 1\rangle\varphi$

can be proved.

1. $\langle 0\rangle\langle 1\rangle[1]\varphi \to \langle 1\rangle\langle 0\rangle[1]\varphi$ (GPerm$_1$)
2. $\langle 0\rangle[1]\varphi \to \langle 1\rangle\langle 0\rangle[1]\varphi$ (S5(1))

3. $\langle 1\rangle\langle 0\rangle[1]\varphi \to \langle 0\rangle\langle 1\rangle[1]\varphi$ (GPerm$_1$)
4. $[1]\langle 1\rangle\langle 0\rangle[1]\varphi \to [1]\langle 0\rangle\langle 1\rangle[1]\varphi$ (K(1))
5. $\langle 1\rangle\langle 0\rangle[1]\varphi \to [1]\langle 0\rangle\langle 1\rangle[1]\varphi$ (S5(1))
6. $\langle 1\rangle\langle 0\rangle[1]\varphi \to [1]\langle 0\rangle[1]\varphi$ (S5(1))
7. $\langle 1\rangle\langle 0\rangle[1]\varphi \to [1]\langle 0\rangle\varphi$ (S5(1))

8. $\langle 0\rangle[1]\varphi \to [1]\langle 0\rangle\varphi$ (From 3 and 7)

LINK WITH PRODUCT LOGIC AND COMPLEXITY

- Hence the logic of the two-agent *Ldm* is nothing but the product $S5^2 = S5 \otimes S5$ [Marx 1999], [Gabbay et al. 2003]
- NEXPTIME-complete.

Fortunately, adding more agents does not lead to a more complex logic:

THEOREM ([BALBIANI, HERZIG, TROQUARD 2008])

(Full) Ldm is NEXPTIME-complete.

COMPLEXITY RESULTS "SEEING TO IT THAT" (SOME POINTERS)

Reasoning about "seeing to it that" is computationally costly.

- Achievement stit: decidable for one-agent case... without busy choosers
- Individual agency Chellas/deliberative stit: NEXPTIME-complete [Balbiani et al. 08]
- Coalitional agency Chellas/deliberative stit: from NEXPTIME-complete to undecidable [Schwarzentruber et al. 07-11]
- Strategic coalitional agency:
    - satisfiability problem: undecidable [Troquard & Walther 12]
    - model checking problem: non-elementary [Brihaye et al. 07-13]

Taming the complexity of coalitional agency:

- Restricting the models: CL-PC [van der Hoek & Wooldridge 05]
- Restricting the language: formulas of "ever growing coalitions" [Schwarzentruber 11], bounded modal depth [Lorini & Schwarzentruber 11]

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