LOGICS OF AGENCY
CHAPTER 4: APPLICATIONS OF AGENCY TO SOCIAL INFLUENCE AND OBLIGATIONS

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ESSLLI 2016 – Bolzano
OVERVIEW OF THIS CHAPTER

- Social influence
- Horty’s obligation to do
More theses from [Belnap et al. 2001]

**Definition (Imperative Content Thesis)**
Regardless of its force on an occasion of use, the content of every imperative is agentive.

**Definition (Restricted Complement Thesis)**
A variety of constructions concerned with agents and agency—including deontic statements, imperatives, and statements of intentions, among others—must take agentives as their complements.

**Definition (STIT Normal Form Thesis)**
In investigations of those constructions that take agentives as complements, nothing but confusion is lost if the complements are taken to be all and only stit sentences.
Some applications for multi-agent systems

Obligations to do
\[ \bigcirc_{John} \triangledown_{John} \text{“dinner is ready”} \]

Delegations / Imperatives:
\[ \triangledown_{Mary} \bigcirc_{John} \triangledown_{John} \text{“dinner is ready”} \]

Constraining a behaviour:
\[ \triangledown_{Mary} \triangledown_{John} \text{“dinner is ready”} \]

Deliberate inaction:
\[ \triangledown_{Mary} \neg \triangledown_{Mary} \text{“Mary eats chocolate”} \]
Outline

1. Social Influence

2. STIT and Deontic Logic
TALKING ABOUT DELEGATION

A common theme: [Chellas 1969], [Santos et al. 1996, 1997], [Norman & Reed 2010]...
Neighbourhoods semantics of individual bringing-it-about (Reminder)

\[ M = \langle W, EE, EC, V \rangle \]

- \( W \) is some set of possible worlds,
- \( V : \text{Prop} \rightarrow \mathcal{P}(W) \) is a valuation function
- \( EE : W \times \text{Agt} \rightarrow \mathcal{P}(\mathcal{P}(W)) \)
- \( EC : W \times \text{Agt} \rightarrow \mathcal{P}(\mathcal{P}(W)) \)

Constraints on the neighborhood functions:

- \( W \notin EE(w, i) \)
- \( \emptyset \notin EC(w, i) \)
- if \( X \in EE(w, i) \), then \( w \in X \)
- if \( X \in EE(w, i) \) and \( Y \in EE(w, i) \) then \( X \cap Y \in EE(w, i) \)
- \( EE(w, i) \subseteq EC(w, i) \)

Truth values:

\[ M, w \models p \quad \text{iff} \quad w \in V(p); \]
\[ M, w \models E_i \varphi \quad \text{iff} \quad \| \varphi \|^M \in EE(w, i); \]
\[ M, w \models C_i \varphi \quad \text{iff} \quad \| \varphi \|^M \in EC(w, i). \]
Neighbourhoods semantics of coalitional bringing-it-about

\[ M = \langle W, EE, EC, V \rangle \]

- \( W \) is some set of possible worlds,
- \( V : \text{Prop} \rightarrow \mathcal{P}(W) \) is a valuation function
- \( EE : W \times \mathcal{P}(\text{Agt}) \rightarrow \mathcal{P}(\mathcal{P}(W)) \)
- \( EC : W \times \mathcal{P}(\text{Agt}) \rightarrow \mathcal{P}(\mathcal{P}(W)) \)

Constraints on the neighborhoud functions:

- analogous to the individual case
- \( EC(w, \emptyset) = \emptyset \)
- \( X \in EE(w, G_1) \) and \( Y \in EE(w, G_2) \) then \( X \cap Y \in EC(w, G_1 \cup G_2) \)

Truth values:

\[
M, w \models p \quad \text{iff} \quad w \in V(p);
\]
\[
M, w \models E_G \varphi \quad \text{iff} \quad \models M \models \varphi \in EE(w, G);
\]
\[
M, w \models C_G \varphi \quad \text{iff} \quad \models M \models \varphi \in EC(w, G).
\]
PRINCIPLES OF COALITIONAL “BRINGING IT ABOUT”

For all groups $G$, $G_1$, and $G_2$ and formulas $\varphi$ and $\psi$:

- **[Ax0]** $\vdash \varphi$, when $\varphi$ is a tautology in propositional logic
- **[Ax1]** $\vdash E_G \varphi \land E_G \psi \rightarrow E_G (\varphi \land \psi)$
- **[Ax2]** $\vdash E_G \varphi \rightarrow \varphi$
- **[Ax3]** $\vdash E_G \varphi \rightarrow C_G \varphi$
- **[Ax4]** $\vdash \neg C_G \bot$
- **[Ax5]** $\vdash \neg C_G \top$
- **[Ax6]** $\vdash \neg C_\emptyset \varphi$
- **[Ax7]** $\vdash E_{G_1} \varphi \land E_{G_2} \psi \rightarrow C_{G_1 \cup G_2} (\varphi \land \psi)$
- **[ERE]** if $\vdash \varphi \leftrightarrow \psi$ then $\vdash E_G \varphi \leftrightarrow E_G \psi$
- **[ERC]** if $\vdash \varphi \leftrightarrow \psi$ then $\vdash C_G \varphi \leftrightarrow C_G \psi$
**Strict Joint Agency**

- It seems right that if $G_1$ acting as a coalition is bringing about a goal $\varphi$, then all the members of $G_1$ are actually contributing in some way to $\varphi$. (e.g., [Lindahl 1977])

- They might be necessary for the performance of a bodily movement, or they might be necessary for the group attitude that is put into the goal $X$.

- We use the term team to capture this.

- The members of the team $G_1$, each with a group attitude towards the coalition $G_1$ with regard to $\varphi$, when $G_2 \subset G_1$ the group $G_2$ cannot be agentive for $\varphi$. At least the group $G_2$ is not bringing about the goal $\varphi$ as a team.

  \[
  [\text{Ax8}] \vdash E_{G_1} \varphi \rightarrow \neg E_{G_2} \varphi, \text{ when } G_2 \subset G_1
  \]

- The contrapositive of the constraint is the notion of strict joint agency ([Belnap et al. 2001], [Carmo 2010]).
Making do / delegation ([Chellas 1969], and many others):

\[ E_a E_b \varphi \]
Responsibility of the Delegator

\[ E_a E_b \varphi \rightarrow E_a \varphi \]

\[ [Ax9] \vdash E_{G_1} E_{G_2} \varphi \rightarrow E_{G_1} \varphi \]

- [Chellas 1969]: “quid facit per alium facit per se”. When agent \( a \) makes another agent bring about something, agent \( a \) is himself bringing about that something.

- Elgesem rejects the constraint: “a person is normally not considered the agent of some consequence of his action if another agent interferes in the causal chain.” [Elgesem 1993, p. 82].

- [Santos et al. 1997] propose two notions of agency:
  - If agent \( a \) directly brings about that \( b \) directly brings about that \( \varphi \) then \( a \) does not directly bring about the \( \varphi \).
  - But they adopt the principle for indirect actions.
**Impossible intra-team commands (I)**

1. $\text{Ax8, Ax9 } \vdash E_a \varphi \rightarrow \neg E_{\{a,b\}} \varphi$ (instance of axiom Ax8)
2. $\text{Ax8, Ax9 } \vdash E_a E_b \varphi \rightarrow E_a \varphi$ (instance of axiom Ax9)
3. $\text{Ax8, Ax9 } \vdash E_a E_b \varphi \rightarrow \neg E_{\{a,b\}} \varphi$ (from 1. and 2. by Propositional Logic)
4. $\text{Ax8, Ax9 } \vdash E_a E_b \varphi \rightarrow E_b \varphi$ (instance of axiom Ax2)
5. $\text{Ax8, Ax9 } \vdash E_a E_b \varphi \rightarrow E_a \varphi \land E_b \varphi$ (from 2. and 4. by PL)
6. $\text{Ax8, Ax9 } \vdash E_a \varphi \land E_b \varphi \rightarrow C_{\{a,b\}} \varphi$ (instance of axiom Ax7)
7. $\text{Ax8, Ax9 } \vdash E_a E_b \varphi \rightarrow \neg E_{\{a,b\}} \varphi \land C_{\{a,b\}} \varphi$ (from 3., 5. and 6. by PL)
IMPOSSIBLE INTRA-TEAM COMMANDS (II)

1. \( Ax8, Ax9 \vdash E_{\{a,b\}} E_b \varphi \rightarrow E_{\{a,b\}} \varphi \) (instance of axiom Ax9)
2. \( Ax8, Ax9 \vdash E_{\{a,b\}} E_b \varphi \rightarrow E_b \varphi \) (instance of axiom Ax2)
3. \( Ax8, Ax9 \vdash E_b \varphi \rightarrow \neg E_{\{a,b\}} \varphi \) (instance of axiom Ax8)
4. \( Ax8, Ax9 \vdash E_{\{a,b\}} E_b \varphi \rightarrow E_{\{a,b\}} \varphi \land \neg E_{\{a,b\}} \varphi \) (from 1., 2., and 3. by PL)
5. \( Ax8, Ax9 \vdash \neg E_{\{a,b\}} E_b \varphi \) (from 4. by PL)

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1. \( Ax8, Ax9 \vdash E_b E_{\{a,b\}} \varphi \rightarrow E_{\{a,b\}} \varphi \) (instance of axiom Ax2)
2. \( Ax8, Ax9 \vdash E_b E_{\{a,b\}} \varphi \rightarrow E_b \varphi \) (instance of axiom Ax9)
3. \( Ax8, Ax9 \vdash E_b \varphi \rightarrow \neg E_{\{a,b\}} \varphi \) (instance of axiom Ax8)
4. \( Ax8, Ax9 \vdash E_b E_{\{a,b\}} \varphi \rightarrow E_{\{a,b\}} \varphi \land \neg E_{\{a,b\}} \varphi \) (from 1., 2., and 3. by PL)
5. \( Ax8, Ax9 \vdash \neg E_b E_{\{a,b\}} \varphi \) (from 4. by PL)
SHARED RESPONSABILITY

\[\text{[Ax10]} \vdash E_{G_1} E_{G_2} \varphi \rightarrow E_{G_1 \cup G_2} \varphi\]

1. \(\text{Ax10} \vdash E_a E_b \varphi \rightarrow E_b \varphi\) (instance of axiom Ax2)
2. \(\text{Ax10} \vdash E_a E_b \varphi \rightarrow E_{\{a,b\}} \varphi\) (instance of axiom Ax10)
3. \(\text{Ax10} \vdash E_a E_b \varphi \rightarrow E_b \varphi \land E_{\{a,b\}} \varphi\) (from 1., and 2. by PL)

Funny interaction with strict joint agency (suppose \(G_2 \neq \emptyset\)):

1. \(\text{Ax10, Ax8} \vdash E_{G_1} E_{G_2} \varphi \rightarrow E_{G_1 \cup G_2} \varphi\) (instance of axiom Ax10)
2. \(\text{Ax10, Ax8} \vdash E_{G_1} E_{G_2} \varphi \rightarrow E_{G_2} \varphi\) (instance of axiom Ax2)
3. \(\text{Ax10, Ax8} \vdash E_{G_2} \varphi \rightarrow \neg E_{G_1 \cup G_2} \varphi\) (instance of axiom Ax8)
4. \(\text{Ax10, Ax8} \vdash \neg E_{G_1} E_{G_2} \varphi\) (from 1., 2., and 3. by PL)
Delegations: commands as imposing norms

General pattern:

\[ E_a \text{OE}_b \varphi \]

Delegations within institutions
[Santos and Jones, Carmo and Pacheco, Sartor, ...]

- agency of an individual agent:

\[ E_a \varphi \Rightarrow \text{“agent } a \text{ is agentive for } \varphi \” \]

- agency of an agent in a role:

\[ E_a : r \varphi \Rightarrow \text{“agent } a \text{ playing the role } r \text{ is (ex-post acto) responsible } \varphi \” \]

- obligations of an agent in a role:

\[ O E_a : r \varphi \Rightarrow \text{“agent } a \text{ playing the role } r \text{ ought to achieve } \varphi \” \]

- institutional delegation:

\[ E_{a: \text{boss}} O E_{b: \text{emp}} \varphi \Rightarrow \text{“Boss } a \text{ delegates } \varphi \text{ to employee } b \” \]
In [Norman & Reed 2010], actions, and doings. E.g.:

- \( E_{Joe} T_{Avrel} \alpha_{\text{shoot}} \)
- \( E_{Joe} E_{Avrel} T_{Avrel} \alpha_{\text{strangle}} \)
“Making do” generally falls flat in STIT theories:

- Chellas’ stit: John makes Ana do $\varphi$ only if $\varphi$ is settled (independence of agents)
- Achievement and deliberative stit: delegation is impossible (negative condition)
- It can be amended nicely in a deontic context (next section of these slides)
Refaining from doing $\varphi$:

\[ [i \textit{dstit} : \neg[i \textit{dstit} : \varphi]] \]

REFREF principle [Belnap et al. 2001]:

\[ [i \textit{dstit} : \neg[i \textit{dstit} : [i \textit{dstit} : \neg[i \textit{dstit} : \varphi]]]] \leftrightarrow [i \textit{dstit} : \varphi] \]

REFREF does not work in astit in general.
OUTLINE

1 SOCIAL INFLUENCE

2 STIT AND DEONTIC LOGIC
Horty’s deontic logic for representing and reasoning about what agents ought to do (and ought to be the case).
**BT STRUCTURES (REMEMBER)**

**BT structure** \( \langle \text{Mom}, < \rangle \):

- History = maximally \(<\)-ordered set of moments
- \( \text{Hist} = \) set of all histories
- \( H_m = \) set of histories passing through the moment \( m \)
- Explode moments into indexe{s} (moment/history pairs)
  - \( m_0/\text{h}_3 \not\models \text{Fp} \)
  - \( m_0/\text{h}_1 \models \text{Fp} \)
A $BT + AC$ model is a tuple $\mathcal{M} = \langle \text{Mom}, <, \text{Choice}, v \rangle$, where:

- $\langle \text{Mom}, < \rangle$ is a $BT$ structure;
- $\text{Choice} : \text{Agt} \times \text{Mom} \rightarrow \mathcal{P}(\mathcal{P}(\text{Hist}))$;
  - $\text{Choice} : \text{Agt} \times \text{Mom} \rightarrow \mathcal{P}(\mathcal{P}(\text{Hist}))$
    - $\text{Choice}(a, m)$ = repertoire of choices for agent $a$ at moment $m$
    - $\text{Choice}$ is a function mapping each agent and each moment $m$ into a partition of $H_m$
  - $\text{Choice}(a, m) : \text{Hist} \rightarrow \mathcal{P}(\text{Hist})$
    - For $h \in H_m$: $\text{Choice}(a, m)(h) = $ the particular choice of $a$ at index $m/h$.
- Independence of agents/choices: Let $h, m$.
  For all collections of $X_a \in \text{Choice}(a, m)(h)$, $\bigcap_{a \in \text{Agt}} X_a \neq \emptyset$.
- No choice between undivided histories: if $\exists m' > m$ s.t. $h, h' \in H_{m'}$ then $h' \in \text{Choice}(a, m)(h)$.
- $v$ is valuation function $v : \text{Prop} \rightarrow \mathcal{P}(\text{Mom} \times \text{Hist})$. 
“Chellas” stit:

\[ M, m/h \models [G\text{ cstit} : \varphi] \iff M, m/h' \models \varphi \text{ for all } h' \in \text{Choice}(G, m)(h) \]

historical necessity:

\[ M, m/h \models \Box \varphi \iff M, m/h' \models \varphi \text{ for all } h' \in H_m \]

Linear time modalities:

- **F** future
- **P** past
A standard deontic models is a tuple \( \mathcal{M} = \langle \text{Mom}, <, \text{Choice}, \text{Ought}, v \rangle \), where:

- \( \langle \text{Mom}, <, \text{Choice}, v \rangle \) is a BT + AC model;
- \text{Ought} maps each moment \( m \in \text{Mom} \) to a set \( \text{Ought}_m \subseteq H_m \).

The set \( \text{Ought}_m \) contains the ideal histories passing through \( m \)

\[
m/h \models \Box \varphi \iff \forall h' \in \text{Ought}_m : m/h' \models \varphi
\]
Standard deontic models sometimes considered too crude: classification as ideal or nonideal.

Instead of ideal (0) / nonideal (1): each $h \in H_m$ is given a value representing the worth or desirability of $h$ at $m$.

The set of values $val$ can be arbitrary (real numbers, color codes, ...) but must be at least partially ordered by $\leq$. 
A general deontic models is a tuple \( \mathcal{M} = \langle \text{Mom}, <, \text{Choice}, \text{Value}, v \rangle \), where:

- \( \langle \text{Mom}, <, \text{Choice}, v \rangle \) is a \( BT + AC \) model;
- \text{Value} maps each moment \( m \in \text{Mom} \) to a function \( \text{Value}_m : H_m \rightarrow \text{val} \).

Let \( m \in \text{Mom} \) and \( h, h' \in H_m \). \( \text{Value}_m(h) \leq \text{Value}_m(h') \) means that \( h' \) is at least as desirable as \( h \) at \( m \).
Let $\mathcal{M} = \langle \text{Mom}, <, \text{Choice}, \text{Value}, v \rangle$.

$\mathcal{M}, m/h \models \Box \varphi \iff \exists h' \in H_m :$

\[
\begin{cases}
(1) & \mathcal{M}, m/h' \models \varphi \\
(2) & \forall h'' \in H_m : \text{if } \text{Value}_m(h') \leq \text{Value}_m(h'') \text{ then } \mathcal{M}, m/h'' \models \varphi
\end{cases}
\]

($\varphi$ is true for some history, and $\varphi$ is true for all histories at least as desirable.)
Utilitarian deontic models are obtained by adding two constraints.

Constraint 1: $\text{val} = \mathbb{R}$, and $\leq$ is the standard order over $\mathbb{R}$

Constraint 2: $\text{Value}_m(h) = \text{Value}_{m'}(h)$

(We can omit the subscript.)
“Reparational” Oughts

Obligations rising from violations of previous obligations. Example ([Thomason 1984], [Horty 2001]):

- Suppose it ought to be the case at the moment $m_1$ that $a$ will soon board a plane to visit his aunt.
- At the moment $m_1$, three histories unfold.
- In $h_1$, he boards the plane.
- In $h_2$, $a$ does not board the plane and calls his aunt to tell her that he will not be visiting.
- In $h_3$, $a$ does not board the plane and does not call his aunt to tell her that he will not be visiting.
- The letter $A$ stands for the proposition that the agent will board the plane; the
- The letter $B$ stands for the proposition that the agent will call his aunt to say that he is not coming.
IDEAL OUGHTS

- $Ought_{m_1} = \{h_1\}$
- $Ought_{m_2} = \{h_2\}$
- $m_1/\_ \models \lozenge A \land \neg \lozenge B$
- $m_2/\_ \models \lozenge B$

(Picture from [Horty 2001])
Utilitarian Oughts

\[ \text{Value}_{m_1}(h_1) = 10 \]
\[ \text{Value}_{m_1}(h_2) = \text{Value}_{m_2}(h_2) = 4 \]
\[ \text{Value}_{m_1}(h_3) = \text{Value}_{m_2}(h_3) = 0 \]
\[ m_1/\_ \models \bigcirc A \land \neg \bigcirc B \]
\[ m_2/\_ \models \bigcirc B \]

(Picture from [Horty 2001])
Originally Meinong, Nicolai Hartmann, ... but also Anderson, Kanger, ...

Roderick Chisholm suggests:

“\( S \) ought to bring it about that \( p \)” can be defined as “It ought to be that \( S \) brings it about that \( p \).”
[Chisholm 1964, p. 150]

Agent \( a \) ought to see to it that \( \varphi \):

\( \Box[a\text{ cstit} : \varphi] \)
\[ h_1 \quad h_2 \quad h_3 \quad h_4 \]

\[ \begin{align*}
A & \quad \neg A \\
\neg A & \quad A
\end{align*} \]

\[ \begin{array}{cc}
K_1 & K_2 \\
\text{m}
\end{array} \]

(Picture from [Horty 2001])

- \( \bigcirc A \)
- \( \neg \bigcirc [a \text{ cstit} : A] \)
Logical principles of utilitarian deontic models

- $\Box$ is a normal modal operator
- $\Box \varphi \rightarrow \Diamond \varphi$
- $\Box \varphi \rightarrow \square \Box \varphi$
- $\neg (\Box [a \text{ cstit} : \varphi] \land \Box [b \text{ cstit} : \neg \varphi])$
- $\Box [a \text{ cstit} : \varphi] \rightarrow \Box \varphi$
  - Remark: $\Box \varphi \rightarrow \Box [a \text{ cstit} : \varphi]$ is not valid
  - Remark: $\Box \varphi \land \Diamond [a \text{ cstit} : \varphi] \rightarrow \Box [a \text{ cstit} : \varphi]$ is not valid either! (next two slides)
Karen, wishes to buy a horse, but she has only $10,000 to spend and the horse she wants is selling for $15,000;

We imagine that Karen offers $10,000 for the horse at the moment $m$ (choice $K_1$);

It is up to the owner of the horse to decide whether to accept the offer. The history $h_1$ represents a scenario in which the owner accepts Karen’s offer, $h_2$ a scenario in which the offer is rejected;

$A$ is the statement that Karen will become less wealthy by the amount of $10,000;

The unique best history is $h_1$, in which the offer is accepted, and, as a consequence, Karen buys the horse and becomes less wealthy by $10,000;

Since Karen is less wealthy by $10,000 in the unique best history, we must conclude that it ought to be that she is less wealthy by $10,000;

Of course, Karen also has the ability to see to it that she is less wealthy by $10,000, (choice $K_2$);

But we would not wish to conclude that Karen ought to see to it that she is less wealthy by $10,000.
- $\Diamond A$
- $\Diamond [a cstit : A]$
- $\neg \Diamond [a cstit : A]$
CRITICISM OF THE UTILITARIAN DEONTIC MODEL

Is $\Box[a cstit : A]$ actually adequate to formalize that an agent $a$ ought to see to it that $A$?

- ... it is entirely utilitarian.
- Horty: a statement that $a$ ought to see to it that $A$ often seems to be sensitive also to nonutilitarian considerations.

Is $\Box[a cstit : A]$ at least adequate to formalize the utilitarian notion of an agent $a$ ought to see to it that $A$?

- An agent $a$ is faced with two options at the moment $m$: to gamble the sum of five dollars ($K_1$), or to refrain from gambling ($K_2$).
- If $a$ gambles, there is a history in which he wins ten dollars, and another in which he loses his stake;
- If $a$ does not gamble, he preserves his original stake;
- the utility associated with each history at $m$ is entirely determined by the sum of money that $a$ possesses at the end;
- The letter $A$ stands for the proposition that $a$ gambles;
- $\Box[a cstit : A]$ holds at $m$. 
(Picture from [Horty 2001])
If we change the utilities:

Then $\neg \bigcirc [\text{acstit} : A]$. Good.
But we should expect that it is wise not to gamble here.
However, $\neg \bigcirc [\text{acstit} : \neg A]$. 
So, with utilitarian deontic models...

The model prescribes risk seeking obligations to the agents. The models do not prescribe “bad” obligations, but they also miss some seemingly reasonable obligations.
A proposition at the moment $m$ is a subset $X \subseteq H_m$.

**Definition** (weak preference over propositions): $X \preceq Y$ iff 
$\forall h \in X, \forall h' \in Y : Value_m(h) \leq Value_m(h')$.

A new “fused” deontic stit operator with ordered choices would be:

$m/h \models \bigoplus [acstit : A] \iff \exists K \in Choice(a, m)$ such that (1) 
$\{m\} \times K \subseteq ||A||$ and (2) $\forall K' \in Choice(a, m) : K' \leq K$. 
Gambling again

$\neg \boxplus[a\ cstit : A] \land \neg \boxplus[a\ cstit : \neg A]$

$\boxplus[a\ cstit : \neg A]$
Further problem with multiagency

K₂ seems preferable, but it is not the case that K₁ ≤ K₂.
**States: choices of others**

We define:

\[
\text{State}(a, m) = \text{Choice}(\text{Agt} \setminus \{a\}, m)
\]

The “strategic contexts” agent \(a\) might face.

When there are two players (e.g., on the previous example):

\[
\text{State}(a, m) = \text{Choice}(b, m)
\]

and

\[
\text{State}(b, m) = \text{Choice}(a, m)
\]
Definition (weak choice dominance): Let $K, K' \in Choice(a, m)$. $K \preceq_a K'$ iff $K \cap S \leq K' \cap S$ for every $S \in States(a, m)$

On the previous example: $K_1 \preceq_a K_2$. 
**Optimal choice**

\[ \text{Optimal}(a, m) = \{ K \in \text{Choice}(a, m) \mid \forall K' \in \text{Choice}(a, m), K \prec_a K' \} \]

When there is a finite number of choices, this works well:

\[
m/h \models \bigcirc[a \text{ cstit} : A] \iff \{m\} \times K \subseteq \|A\| \quad \text{for every} \quad K \in \text{Optimal}(a, m)
\]
Further problem with infinite repertoires of choices

We’d like to have $\Box[a\text{ cstit} : A]$ and $\neg \Box[a\text{ cstit} : \neg A]$. But $Optimal(a, m) = \emptyset$...
$m/h \models \bigcirc [a \text{ cstit} : A]$ iff for every $K \in \text{Choice}(a, m)$, if 
$
\{m\} \times K \not\subseteq \|A\|,
$
then there is $K' \in \text{Choice}(a, m)$ such that:

(1) $K \prec_a K'$, and

(2) $\{m\} \times K' \subseteq \|A\|$, and

(3) $\{m\} \times K'' \subseteq \|A\|$ for each $K'' \in \text{Choice}(a, m)$ such that $K' \preceq_a K''$.

This is obligation to do.
 SOCIAL INFLUENCE: COMMANDS

What about?

\[ a \text{ cst}it : \bigcirc [b \text{ cst}it : \phi] \]

\[ a \text{ cst}it : \bigcirc F [b \text{ cst}it : \phi] \]

\[ a \text{ cst}it : F \bigcirc [b \text{ cst}it : \phi] \]

\[ a \text{ cst}it : \bigodot [b \text{ cst}it : \phi] \]

\[ a \text{ cst}it : \bigodot F [b \text{ cst}it : \phi] \]

\[ a \text{ cst}it : F \bigodot [b \text{ cst}it : \phi] \]
SOCIAL INFLUENCE: COMMANDS

Well:

- \([a \text{ cstit} : \bigcirc [b \text{ cstit} : \varphi]]\) is just \(\bigcirc [b \text{ cstit} : \varphi]\)
- \([a \text{ cstit} : \bigcirc F [b \text{ cstit} : \varphi]]\) is just \(\bigcirc F [b \text{ cstit} : \varphi]\)
- \([a \text{ cstit} : F \bigcirc [b \text{ cstit} : \varphi]]\) is good
- \([a \text{ cstit} : \bigcirc [b \text{ cstit} : \varphi]]\) is just \(\bigcirc [b \text{ cstit} : \varphi]\)
- \([a \text{ cstit} : \bigcirc F [b \text{ cstit} : \varphi]]\) is not in our language!
- \([a \text{ cstit} : F \bigcirc [b \text{ cstit} : \varphi]]\) is good
Restricted complement thesis $\Rightarrow$ deontic statements must take agentives as their complements.

Some arguments against the simple logic for $\circ$.

It worked with $\circ[a cstit : \varphi]$, but it is a fused operator, not exactly an ought with an agentive in its scope. Nonetheless, it shows that the models are amenable.
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