ESSLLI Tutorial: Nonmonotonic Logic

Structured Argumentation

Mathieu Beirlaen[1]   Christian Straßer[1,2]

August 26, 2016

[1]Institute for Philosophy II, Ruhr-University Bochum in

[2]Center for Logic and Philosophy of Science, Ghent University

Aims of this session

- learn about the basic ideas behind Structured Argumentation
- learn about how to handle priorities
- learn about some possible pitfalls

On the way to Structured Argumentation

Formal Argumentation as a Model for Defeasible Reasoning

- reasoning as an argumentative activity an agent has with herself

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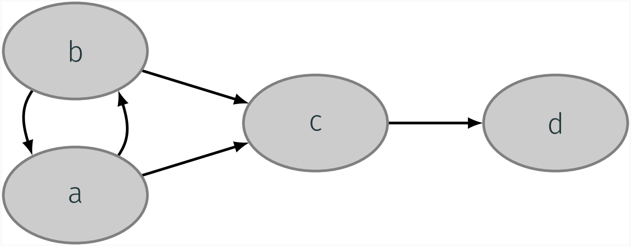
Formal Argumentation as a Model for Defeasible Reasoning

- reasoning as an argumentative activity an agent has with herself
- defeasibility as a result of the dynamics that results from tensions between considerations and counter-considerations
- some empirical evidence for the material adequacy of such a formal account Mercier and Sperber (2011)

Shifting Perspective: from Support to Attack and Acceptability Dung (1995)



- argument: abstract, points in a directed graph
- arrows: argumentative attacks

Argumentation Semantics

select sets of arguments that represent *rational stances*, i.e., they are conflict-free, defended, etc.

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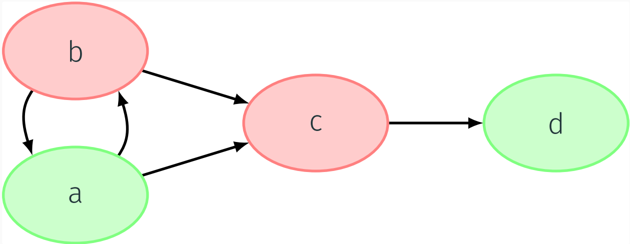

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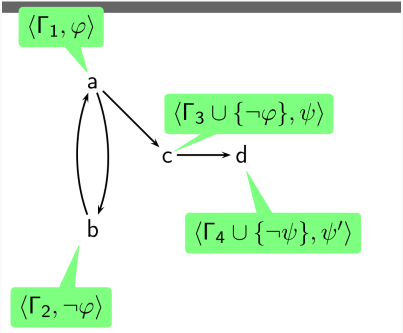
Back to Formal Logic: Structural / Instantiated Argumentation



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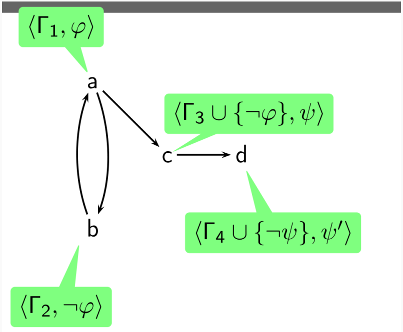

- structured arguments

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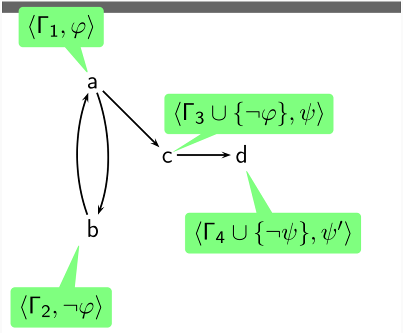

- structured arguments
- define attacks relative to this structure

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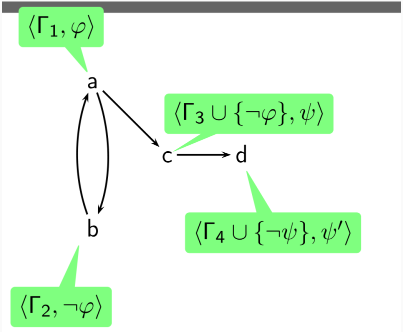

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    - rebuttal

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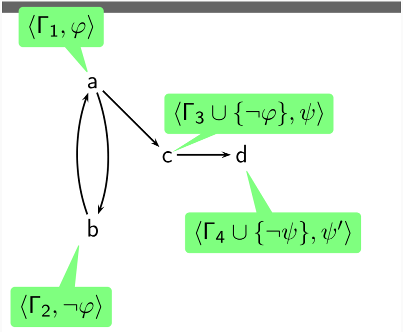

- structured arguments
- define attacks relative to this structure
    - rebuttal
    - premise-attack (sometimes ‘undercut’)

Some of the proposed systems (non-exhaustive)

Dung-based

- ASPIC$^+$ Prakken (2011); Modgil and Prakken (2013, 2014)
- ABA (Assumption-Based Argumentation) Dung et al. (2009)
- Logic-Based Argumetation Besnard and Hunter (2001, 2009)
- Sequent-based Argumentation Arieli (2013); Arieli and Straßer (2015)

Some of the proposed systems (non-exhaustive)

Not Dung-based (doesn't mean not Dung-related)

- OSCAR: Pollock (1995)
- Defeasible Logic: Nute (1994); Governatori et al. (2004)
- Defeasible Logic Programming: García and Simari (2004)
- DEFLOG: Verheij (2000, 2003)
- etc.

What are arguments in ASPIC$^+$?

Rules and Argumentation Systems

- In ASPIC$^+$ we deal with two types of rules:
    1. strict rules, written: $A_1, \ldots, A_n \rightarrow B$
    2. defeasible rules, written: $A_1, \ldots, A_n \Rightarrow B$

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Definition 1 (Argumentation System)

An argumentation system $\mathrm{AS} = \langle \mathcal{L}, \mathcal{S}, \mathcal{D}, \bar{\ } \rangle$ in a formal language $\mathcal{L}$ consists of a set of strict rules $\mathcal{S}$, a set of defeasible rules $\mathcal{D}$, and a contrariness function from $\mathcal{L}$ to $2^{\mathcal{L}}$.

think about negation for now

Knowledge base

Arguments are built on top of a knowledge base. We have two types of information in our knowledge base:

- strict/certain information collected in the set $\mathcal{K}_n$
- assumptions: collected in the set $\mathcal{K}_a$

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Definition 2 (Knowledge Base)

A knowledge base is a set $\mathcal{K}$ of formulas (in $\mathcal{L}$) where $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_a$ and $\mathcal{K}_n \cap \mathcal{K}_a = \emptyset$.

Definition 3 (Arguments)

Let $\text{AS} = \langle \mathcal{L}, \mathcal{S}, \mathcal{D}, \overline{} \rangle$ be an argumentation system and $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_a$ a knowledge base. An argument a based on AS and $\mathcal{K}$ is:

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  $\mathrm{Sub}(a) = \{a\}$
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- $\langle a_1, \ldots, a_n \mapsto B \rangle$ where $a_1, \ldots, a_n$ are arguments and there is a strict rule $\mathrm{conc}(a_1), \ldots, \mathrm{conc}(a_n) \rightarrow B \in \mathcal{S}$

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  $\mathrm{DefRules}(a) = \mathrm{DefRules}(a_1) \cup \ldots \cup \mathrm{DefRules}(a_n) \cup \{r\}$

Arguments

We write $\mathrm{Arg}(\mathsf{AS}, \mathcal{K})$ for the set of all arguments built on top of AS and $\mathcal{K}$.

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We can construct, among others, the following:

- $a_1 = \langle \neg q \rangle$ and $a_2 = \langle \neg q \rangle$ (since $q, \neg q \in \mathcal{K}$)

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- defeasible argument: at least one defeasible rule is used
- firm argument: only based on strict premises in $\mathcal{K}_n$
- plausible argument: at least one defeasible premise in $\mathcal{K}_a$ is used

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We come back to this in a slide …

Definition 4 (Argumentative Attack)

Where $a, b \in \mathrm{Arg}(\mathsf{AS}, \mathcal{K})$,

- a undermines b iff $\mathrm{Conc}(a) \in \overline{B}$ for some $B \in \mathrm{Prem}(b)$

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- a (restricted) rebuts b iff $\mathrm{Conc}(a) \in \overline{B}$ where $B = \mathrm{Conc}(b')$ for some $b' \in \mathrm{Sub}(b)$ and b' is of the form $\langle b_1, \ldots, b_m \Rrightarrow B \rangle$

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- a (restricted) rebuts b iff $\mathrm{Conc}(a) \in \overline{B}$ where $B = \mathrm{Conc}(b')$ for some $b' \in \mathrm{Sub}(b)$ and b' is of the form $\langle b_1, \ldots, b_m \Rrightarrow B \rangle$
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- a undercuts b iff $\mathrm{Conc}(a) = \overline{N(r)}$ for some $b' \in \mathrm{Sub}(b)$ where b' is of the form $\langle b_1, \ldots, b_m \Rrightarrow B \rangle$ and based on the defeasible rule $r = \mathrm{Conc}(b_1), \ldots, \mathrm{Conc}(b_n) \Rightarrow B$ with the name $N(r)$.

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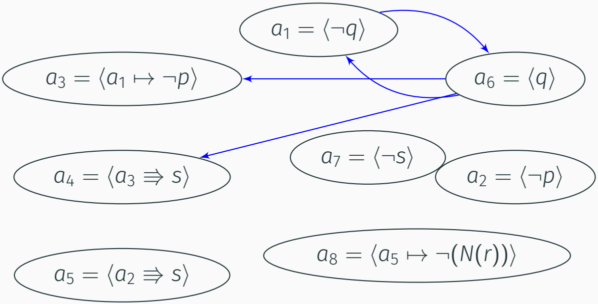
So $\mathrm{penguin} \in \overline{{\sim}\mathrm{penguin}}$. But $\langle {\sim}\mathrm{penguin} \rangle$ should not attack an argument with the conclusion $\mathrm{penguin}$. So, ${\sim}\mathrm{penguin} \notin \overline{\mathrm{penguin}}$.

Structured Argumentation System (without priorities)

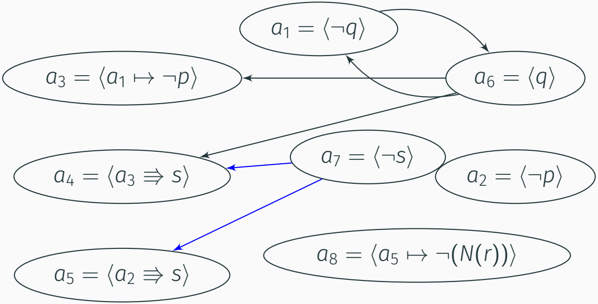
A structured argumentation system $\mathsf{AT} = \langle \mathrm{Arg}(\mathsf{AS}, \mathcal{K}), \rightsquigarrow \rangle$ is an argumentation system equipped with argumentative attacks (define in some, possibly all, of the above ways) giving rise to $\rightsquigarrow \subseteq \mathrm{Arg}(\mathsf{AS}, \mathcal{K}) \times \mathrm{Arg}(\mathsf{AS}, \mathcal{K})$.

Back to the example



- undermining

Back to the example



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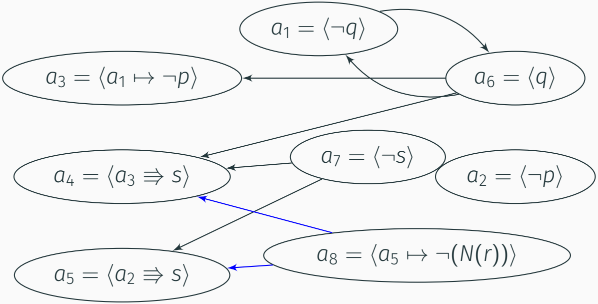

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Argumentation Semantics

We use Dung-style semantics to select sets of arguments. A set $\mathcal{B} \subseteq \mathrm{Arg}(\mathsf{AS}, \mathcal{K})$

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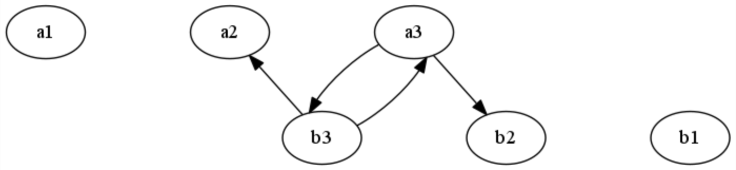
giving rise to (with restricted rebuts)



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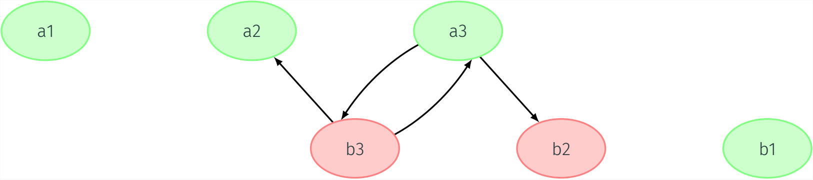

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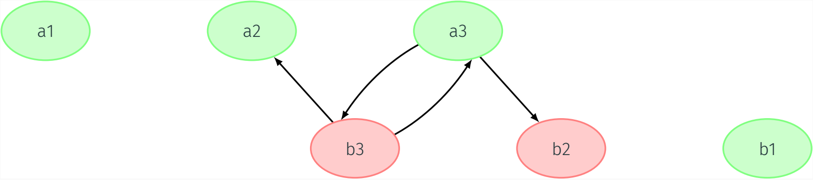

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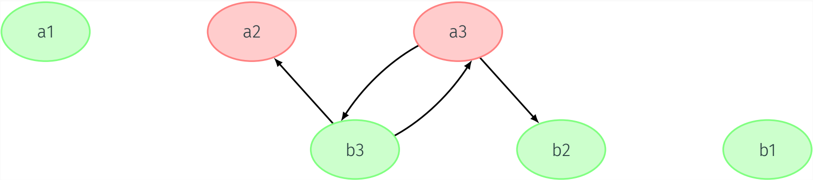

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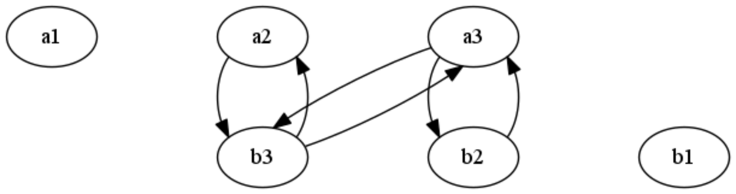
and now we allowing for rebuts on conclusions obtained by strict rules:



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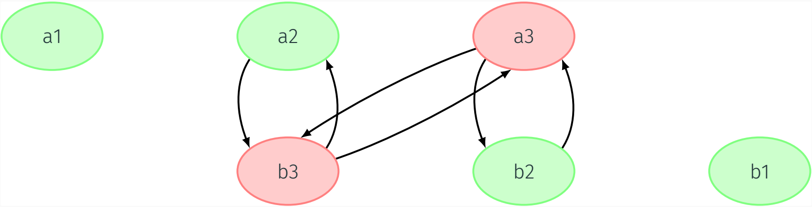
Problem: now we also get the preferred extension:



Definition 5

Where $\mathsf{AT} = \langle \mathrm{Arg}(\mathsf{AS}, \mathcal{K}), \rightsquigarrow \rangle$ is a structured argumentation framework and the semantics sem is one of the Dung-semantics defined above, we define:

- $\mathsf{AT} \mid\!\sim^{\cup}_{\mathsf{sem}} A$ iff there is an $a \in \mathcal{B}$ with $\mathrm{Conc}(a) = A$ for some $\mathcal{B} \in \mathsf{sem}(\mathsf{AT})$

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Note: $\mathrel{|\!\sim}^{\cap}_{\mathsf{sem}}$ admits floating conclusions, while $\mathrel{|\!\sim}^{\Cap}_{\mathsf{sem}}$ blocks them.

Example: Nixon

- $\mathcal{K}_a = \{\mathrm{quaker}, \mathrm{republican}\}$
- $\mathcal{D}$ consists of
  - $\mathrm{quaker} \Rightarrow \mathrm{dove}$
  - $\mathrm{republican} \Rightarrow \mathrm{hawk}$
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We have three preferred extensions (highlighted arguments for polMotivated),

- one including a_1, a_2, a_3
- one including a_4, a_5, a_6
- one including a_1, a_3, a_4, a_6

Introducing priorities

On the level of arguments

Definition 6 (Structured Argumentation Framework)

A structured argumentation framework $\mathsf{AT} = \langle \mathrm{Arg}(\mathsf{AS}, \mathcal{K}), \rightsquigarrow, \preceq \rangle$ where AS is an argumentation theory, $\mathcal{K}$ a knowledge base, $\rightsquigarrow \subseteq \mathrm{Arg}(\mathsf{AS}, \mathcal{K}) \times \mathrm{Arg}(\mathsf{AS}, \mathcal{K})$ an attack relation and $\preceq \subseteq \mathrm{Arg}(\mathsf{AS}, \mathcal{K}) \times \mathrm{Arg}(\mathsf{AS}, \mathcal{K})$ an preorder (reflexive and transitive) on $\mathrm{Arg}(\mathsf{AS}, \mathcal{K})$.

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Where $a, b \in \mathrm{Arg}(\mathsf{AS}, \mathcal{K})$, a defeats b iff $a \rightsquigarrow b$ and

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We can now define the argumentation semantics relative to the notion of defeat instead of the notion of argumentative attack.

Example

Suppose we have:

- $\mathcal{K}_a = \{\mathrm{WearsRing}, \mathrm{PartyAnimal}\}$
- $\mathcal{D} = \{r_1 = \mathrm{WearsRing} \Rightarrow \mathrm{Married}, r_2 = \mathrm{PartyAnimal} \Rightarrow \mathrm{Bachelor}\}$
- $\mathcal{S} = \{\mathrm{Married} \rightarrow \neg\mathrm{Bachelor}, \mathrm{Bachelor} \rightarrow \neg\mathrm{Married}\}$

... and we have the arguments:

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Example

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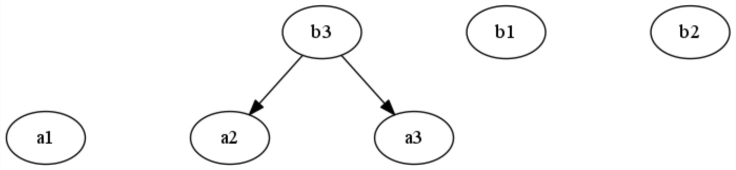
Where $a_2, a_3 \prec b_2, b_3$ then we have the following defeat graph:



Calculating Argument Strength bottom-up

Suppose we are equipped with priority ordering $\leq \subseteq (\mathcal{K}_a \times \mathcal{K}_a) \cup (\mathcal{D} \times \mathcal{D})$ on

- the defeasible premises $\mathcal{K}_a$ and
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We are interested in lifting this on the level of arguments constructed using information in $\mathcal{K}_a \cup \mathcal{K}_n$ and rules in $\mathcal{D} \cup \mathcal{S}$.

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We introduce two possible ways of doing so, via

1. the weakest-link principle
2. the last-link principle

The idea behind weakest-link is that an argument is as strong as its weakest link, which can be a used assumption in $\mathcal{K}_a$ or a used defeasible rule in $\mathcal{D}$.

We first lift $\leq$ to sets of formulas:

Definition 8 (Elitist Lifting, from $\leq$ to $\trianglelefteq$)

Where $\Xi, \Xi' \in 2^{\mathcal{K}_a} \cup 2^{\mathcal{D}}$ are finite,

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Definition 9 (Weakest Link Ordering, from $\trianglelefteq$ to $\preceq$)

Where $a, b \in \mathrm{Arg}(\mathsf{AS}, \mathcal{K})$, $a \preceq b$ iff

1. if both a and b are strict, then $\mathrm{Prem}(a) \cap \mathcal{K}_a \trianglelefteq \mathrm{Prem}(b) \cap \mathcal{K}_a$

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2. if both a and b are firm, then $\mathrm{DefRules}(a) \trianglelefteq \mathrm{DefRules}(b)$.
3. else: $\mathrm{Prem}(a) \cap \mathcal{K}_a \trianglelefteq \mathrm{Prem}(b) \cap \mathcal{K}_a$ and $\mathrm{DefRules}(a) \trianglelefteq \mathrm{DefRules}(b)$

The rationale behind last-link is that arguments are compared in their last link. As a result, an argument a is preferred over b if its last used defeasible rules are preferred over the last defeasible rules used in b.

Definition 10 (Last defeasible rules)

Where a is a defeasible argument:

- if $a = \langle a_1, \ldots, a_n \Rrightarrow A \rangle$ then $\mathrm{LastDefRules}(a) = \{\mathrm{Conc}(a_1), \ldots, \mathrm{Conc}(a_n) \Rightarrow A\}$

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Where $a, b \in \mathrm{Arg}(\mathsf{AS}, \mathcal{K})$, then $a \preceq b$ iff

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1. a is a defeasible argument and b a strict argument, or
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3. $\mathrm{Prem}(a) \cap \mathcal{K}_a \trianglelefteq \mathrm{Prem}(b) \cap \mathcal{K}_a$ if both are strict arguments.

Remark: Lifting

Instead of using the elitist lifting, one may also consider the democratic lifting principle, according to which:

Definition 12 (Democratic Lifting)

Where $\Xi, \Xi' \in 2^{\mathcal{K}_a} \cup 2^{\mathcal{D}}$ are finite,

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… time for the snoring professor …

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We are interested in the conflict between a_3 and a_5.

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- $\text{LastDefRules}(a_3) = r_2$
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Since $r_3 < r_2$ we have $a_5 \prec a_3$ and so a_3 strictly defeats a_5.

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- $\text{Prem}(a_3) \cap \mathcal{K}_a = \{\text{snores}\} \trianglelefteq \{\text{prof}\} = \text{Prem}(a_5) \cap \mathcal{K}_a$

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- $\text{Prem}(a_3) \cap \mathcal{K}_a = \{\text{snores}\} \trianglelefteq \{\text{prof}\} = \text{Prem}(a_5) \cap \mathcal{K}_a$
- $\text{DefRules}(a_3) = \{r_1, r_2\} \trianglelefteq \{r_3\} = \text{DefRules}(a_5)$

Changing the Interpretation to an epistemic one

Suppose we have:

- $\mathcal{K}_a = \{\text{bornInScotland}, \text{fitnessLover}\}$
- $\mathcal{D}$ consists of
  - $r_1 = \text{bornInScotland} \Rightarrow \text{Scottish}$
  - $r_2 = \text{Scottish} \Rightarrow \text{likesWhisky}$
  - $r_3 = \text{fitnessLover} \Rightarrow \neg\text{likesWhisky}$
- $\text{bornInScotland} < \text{fitnessLover}$ and $r_1 < r_3 < r_2$, $r_1 < r_2$

We have, e.g., the following:

- $a_1 = \langle \text{bornInScotland} \rangle$
- $a_2 = \langle a_1 \Rrightarrow \text{Scottish} \rangle$
- $a_3 = \langle a_2 \Rrightarrow \text{likesWhisky} \rangle$
- $a_4 = \langle \text{fitnessLover} \rangle$
- $a_5 = \langle a_4 \Rrightarrow \neg\text{likesWhisky} \rangle$

Now it seems more reasonable to go with Weakest-Link!

Rationality Postulates

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4. indirect consistency: the set obtained by closing $\{\mathrm{Conc}(a) \mid a \in \mathcal{E}\}$ under the strict rules in $\mathcal{S}$ is consistent.

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   - contraposition: for all $\Xi \subseteq \mathcal{L}$ and $A \in \Xi$, if $\Xi \vdash_{\mathcal{S}} B$ then $\Xi \setminus \{A\} \cup \{B'\} \vdash_{\mathcal{S}} A'$ where B' is a contrapositary of B and A' is a contrapositary of A

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Note: the Weakest/Last-Link principles as defined above are reasonable.

Why Transposition

Suppose we use:

- $\mathcal{K}_a = \{\mathrm{WearsRing}, \mathrm{PartyAnimal}\}$
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Why well-formedness?

Recall: whenever A is the contrary of some B, B is not a strict premise or the consequent of a strict rule.

We come back to our application with negation-as-failure. Suppose we have:

- $\mathcal{K}_n = \{\sim\text{penguin}, \text{livesInAlaska}, \text{bird}\}$
- $\mathcal{K}_a = \emptyset$
- $\mathcal{D} = \{\text{bird}, \text{livesInAlaska} \Rightarrow \text{penguin}\}$

Can you see why direct consistency doesn't hold for this example?

Contamination, Interference Wu (2012); Caminada et al. (2012)

Another rationality postulate is Non-Interference: it says that for two sets of formulas Ξ and Ξ' that are syntactically disjoint (they share no atoms) we have $Cn(\Xi)_{|\mathrm{Atoms}(\Xi)} = Cn(\Xi \cup \Xi')_{|\mathrm{Atoms}(\Xi)}$.

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In particular, there should not be a contaminating set, that is a set Λ (where $\mathrm{Atoms}(\Lambda) \subset \mathrm{Atoms}(\mathcal{L})$) such that for every Ξ that is syntactically disjoint from Λ, $Cn(\Lambda) = Cn(\Lambda \cup \Xi)$.

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- suppose our strict rules allow for all the inferences of classical logics, in particular $s, \neg s \rightarrow \neg r$

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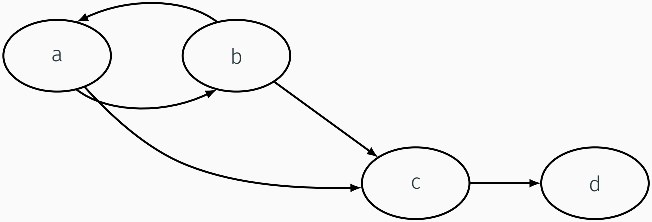

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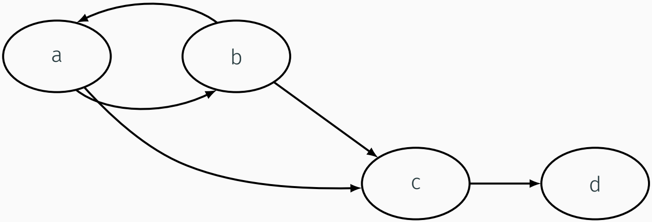

Exercise

- See what happens in grounded semantics!
- Does it help to move to e.g., preferred semantics?

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- $a_2 = \langle\langle\mathrm{Js}\rangle, \langle\mathrm{Jrel}\rangle \Rrightarrow s\rangle$
- $b_1 = \langle\langle\mathrm{Munr}\rangle, \langle\mathrm{Mrel}\rangle \Rrightarrow \neg\mathrm{Mrel}\rangle$
- $b_2 = \langle\langle\mathrm{Mns}\rangle, \langle\mathrm{Mrel}\rangle \Rrightarrow \neg s\rangle$
- $c = a_2, b_2 \mapsto \neg r$

A complication

- $\mathcal{K}_a = \{\mathrm{Js}, \mathrm{Jrel}, \mathrm{Mns}, \mathrm{Mrel}, \mathrm{Wr}, \mathrm{Wrel}, \mathrm{Junr}, \mathrm{Munr}\}$,
- $\mathcal{D}$ consists of
  - $\mathrm{Wr}, \mathrm{Wrel} \Rightarrow r$
  - $\mathrm{Junr}, \mathrm{Jrel} \Rightarrow \neg\mathrm{Jrel}$ (If John says he's unreliable and he's reliable, then he's unreliable.)
  - $\mathrm{Munr}, \mathrm{Mrel} \Rightarrow \neg\mathrm{Mrel}$
  - $\mathrm{Js}, \mathrm{Jrel} \Rightarrow s$
  - $\mathrm{Mns}, \mathrm{Mrel} \Rightarrow \neg s$

We have, for instance, the following arguments:

- $a_1 = \langle\langle\mathrm{Junr}\rangle, \langle\mathrm{Jrel}\rangle \Rrightarrow \neg\mathrm{Jrel}\rangle$
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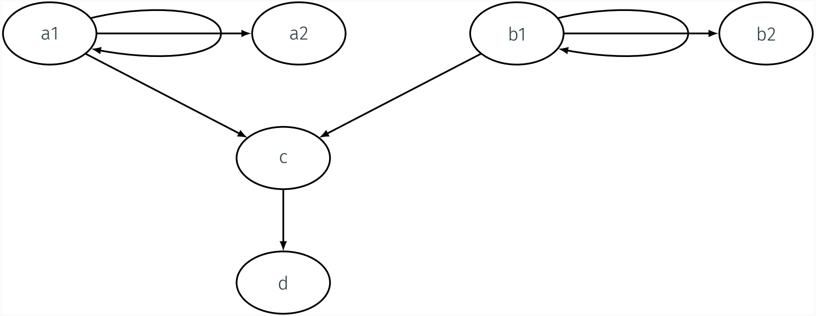
- $c = a_2, b_2 \mapsto \neg r$
- $d = \langle\langle\mathrm{Wr}\rangle, \langle\mathrm{Wrel}\rangle \Rrightarrow r\rangle$

A complication (cont.)

- $a_1 = \langle\langle \mathrm{Junr}\rangle, \langle \mathrm{Jrel}\rangle \Rrightarrow \neg \mathrm{Jrel}\rangle$
- $a_2 = \langle\langle \mathrm{Js}\rangle, \langle \mathrm{Jrel}\rangle \Rrightarrow s\rangle$
- $b_1 = \langle\langle \mathrm{Munr}\rangle, \langle \mathrm{Mrel}\rangle \Rrightarrow \neg \mathrm{Mrel}\rangle$
- $b_2 = \langle\langle \mathrm{Mns}\rangle, \langle \mathrm{Mrel}\rangle \Rrightarrow \neg s\rangle$
- $c = \langle a_2, b_2 \mapsto \neg r\rangle$
- $d = \langle\langle \mathrm{Wr}\rangle, \langle \mathrm{Wrel}\rangle \Rrightarrow r\rangle$



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