# Logics on words and trees with data

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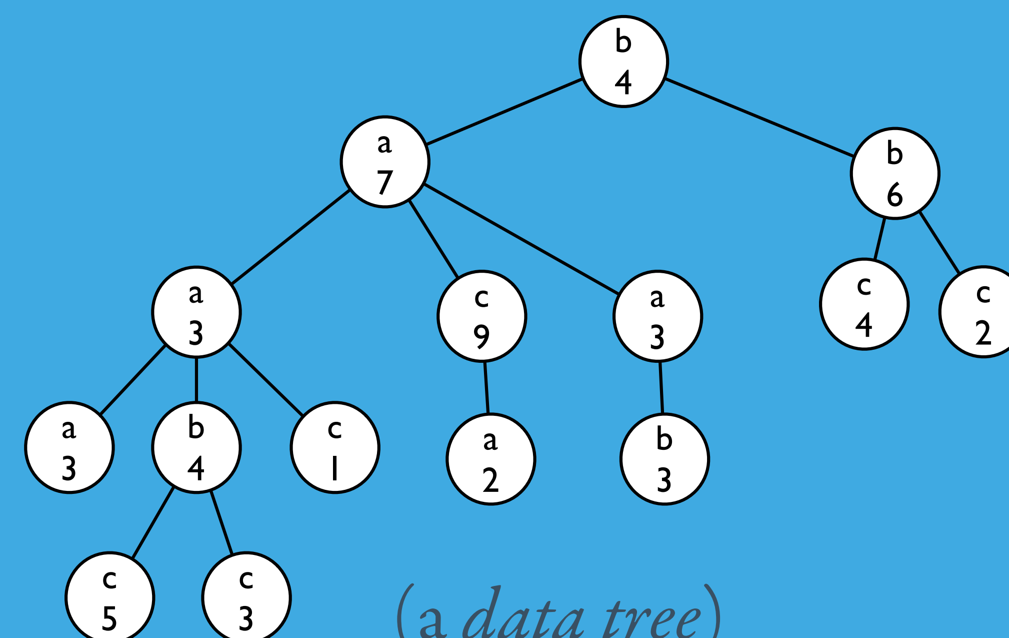
# words / trees with infinite & finite alphabets

## data words

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *a* | *b* | *a* | *b* | *a* | *b* | *b* | *b* | *b* | *a* | *b* | *b* |
| 2 | 4 | 7 | 6 | 9 | 3 | 9 | 7 | 6 | 2 | 2 | 9 |

(a *data word*)

## data trees




(a *data tree*)

questions

How to reason about these structures?

What properties are hard/easy?

Decidability bounds?

Connections with other areas?

# Agenda

- Monday (Diego, Ranko): Introduction, data words
- Tuesday (Ranko): Data words, first-order logic
- Wednesday (Ranko): Data words, temporal logics
- Thursday (Diego): Data trees, path-based logics
- Friday (Diego): Data trees, other formalisms

# PLEASE ASK

(easy)

# QUESTIONS

*data words*

# *data words*

*data word* $\begin{array}{cccccccccccc} a & b & a & b & a & b & b & b & b & a & b & b \\ 2 & 4 & 7 & 6 & 9 & 3 & 9 & 7 & 6 & 2 & 2 & 9 \end{array} \in (\{a,b\} \times \mathbb{N})^*$

*“for every b there is a b with same data value to its right”*

$\begin{array}{ccccccc} r(\mathrm{A}) & r(\mathrm{B}) & w(\mathrm{A}) & w(\mathrm{B}) & commit & r(\mathrm{A}) & w(\mathrm{A}) \\ 4 & 7 & 4 & 7 & 4 & 7 & 7 \end{array}$

*“between r(A) and commit of process P there are no w(A) from any other process P’”*

*What does it represent?*

*execution of concurrent processes,*

*usage of some unbounded resources,*

*timed words,*

*temporal databases,*

*runs of counter automata (or inf. state aut.),*

*. . .*

# Reasoning on data-words

Given a logic $\mathscr{L}$ on data-words,

**Satisfiability problem**

**Input:** $\phi \in \mathscr{L}$
**Output:** is there a data-word w so that $w \models \phi$ ?

**Implication problem**

**Input:** $\phi, \psi \in \mathscr{L}$
**Output:** is is true that $w \models \phi$ implies $w \models \psi$ for every data word w ?

If $\mathscr{L}$ closed under boolean operators:

- implication$(\phi, \psi) \equiv \neg$ sat$(\phi \wedge \neg\psi)$
- sat$(\phi) \equiv \neg$ implication$(\phi, \bot)$

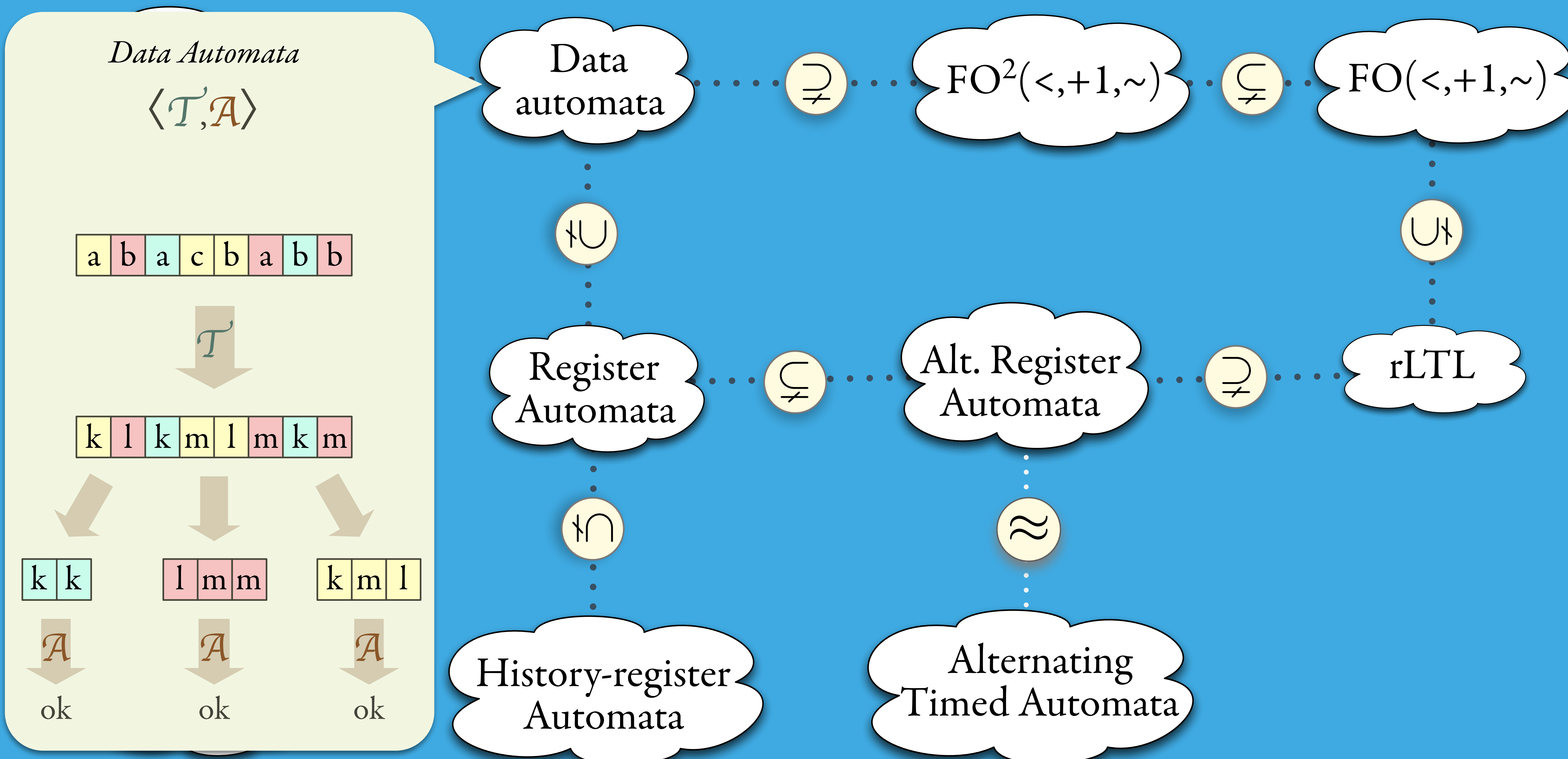
# Why *reasoning*?

- It's fun! 😀
- Basic question for understanding a formalism: Does this mean anything at all? Is this a property?
- Query optimisation:
  - If Q ≡ Q' then it is "safe" to replace Q with a more efficient Q'
  - If Q is unsatisfiable (it contains a contradiction): its computation can be avoided
- In general: verify statically whether a program satisfies a specification (eg, query accessing forbidden info)

# Proviso

- We consider **closure under isomorphism** of data values (ie, only equality/inequality)
- We will mostly focus on **finite** structures
- We will mostly focus on **logics** (closed under boolean connectives)
- **Boolean** formulas (ie, 'properties' instead of 'queries')

# A *zoo* of formalisms on data words



# reasoning with *infinite* alphabets ≈ counting

*there must be 3 distinct data values to the right*

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *a* | *b* | *a* | *b* | *a* | *b* | *b* | *b* | *b* | *a* | *b* | *b* |
| 2 | 4 | 7 | 6 | 9 | 3 | 9 | 7 | 6 | 2 | 2 | 9 |

*"for every **a** there is a **b** with same data value to its right"*

# Counting automata!

# Counter systems

- Minsky machine
  *machine with counters and test for zero*
- Counter machine
  *we don't allow test for zero*
- Gainy counter machine
  *the machine broke down! Increments along the run*

# Counter systems

Reachability problem

*is there a computation ending with counters in 0?*

Control-state reachability problem

*is there one ending with a given state?*

**Wait! Why are we talking about counter automata?!**

Is this ESSLLI?

Am I in the course about data logics..?

**logics on data words**

$\approx$

**counter automata**

Oh! Allright... (?)

Who's that guy?

# Minsky Machine

- Minsky Machine = non-det. finite automata + **counters**
- A counter can only store a natural number ($\geq 0$)
- Operations on counters
  - Check if counter if zero
  - Increment counter by one
  - Decrement counter by one (only if $\neq 0$)

# Minsky Machine

- $\mathcal{A} = (Q, q_0, \delta, k)$, automaton with $k$ counters over finite statespace Q
- Instructions: $\delta \subseteq Q \times \{\text{inc},\text{dec},\text{tz}\} \times \{1,\ldots,k\} \times Q$
- Configurations: $c \in Q \times \mathbb{N}^k$ *eg*: (q,(3,0,2))
- Run: defined by relation $(q, v) \rightsquigarrow (q',v')$ if there is $(q, \textbf{inst}, \textbf{i}, q') \in \delta$ so that v' is the result of applying instruction **inst** to counter **i**.

  *eg*: $(q,(3,0,2)) \rightsquigarrow (q',(2,0,2))$ using $(q,\text{dec}(1),q') \in \delta$.

# Minsky Machine

- Example: $\mathcal{A} = (\{q_0,q_1\},q_0,\delta, 2)$, where

$$\delta = \{(q_0, \text{inc}(1), q_1), (q_1, \text{inc}(2), q_0)\}.$$

- A possible run:

$$(q_0,(0,0)) \rightsquigarrow (q_1, (1,0)) \rightsquigarrow (q_0, (1,1)) \rightsquigarrow (q_1, (0,0)) \rightsquigarrow \cdots$$

# Reachability problems

- **Reachability**: Given a counter automaton $\mathcal{A}$ and a configuration (q,v): is there a run leading to (q,v)
- **Control-state reachability**: Given a counter automaton $\mathcal{A}$ and a state q: is there a run leading to (q,v) **for some v**?

# Reachability problems

Control-state reachability for Minsky Machines is **undecidable**, already for two counters.

*What about reachability?*

# Reachability problems

2-counter Minsky machines are Turing-complete:

- ★ A TM can be simulated by two stacks (infinite tape is cut in half)
- ★ A stack can be simulated by two counters (one of the counters is the binary representation of the bits on the stack)
- ★ Four counters can be simulated by two counters (factorization of one of the counters is $2^a 3^b 5^c 7^d$)

# Decidable restrictions

Two basic ways of turning Minsky Machines into a decidable model:

1. **no tests for zero**, or
2. allow a "faulty" behaviour, where counters can **non-deterministically increment** their value.

# Counter Machine

- A **counter machine** = A Minsky machine without zero tests.
- Equivalent to: Vector Addition Systems (VAS), Petri Nets.
- Reachability and control-state reachability problems are **decidable**.
- Best bound for reachability: non-primitive recursive (hard proof).

  [Sacerdote, Tenney, Mayr, Kosaraju, ...]
- Complxity of control-state reachability: ExpSpace-complete.

  [Rackoff, Lipton]

# Gainy Counter Machine

- It is defined as a Minsky Machine but inside a run there can be **non-deterministic increments** to any counter.
- Reachability / control-state reachability for Gainy Counter Machines is **decidable**, with (provably) non-primitive recursive complexity. [Schoebelen,Abdulla&Jonsson, Finkel&Cécé&Iyer]

Minsky machine

Counter machine

Gainy counter machine

Reachability problem

Control-state reachability problem

Satisfiability problem for data logics