Simplicity of Meaning

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§1. Questions

1. What is the meaning of “walk” as opposed to “run”? Of “left” as opposed to “above”? Of “beech” as opposed to “elm”?
2. How do we tell one from the other?
3. Why are the descriptions so complicated?
4. Why is that not interesting to formal semantics?

§2. Part 1. Models

1. *Outer model*: aka reality.
2. *Inner model*: our representation of the first

$$\text{Code} \xrightarrow{\text{realisation}} \text{Object}$$

- The realisation is a function. It is *implicit*, since it is not part of either model.
- As the mind grasps an object via its code, it connects to reality via the realisation. (Embodiment of meanings.)

§3. More Models

- But is there only one inner model? Couldn't there be several?
- Consider a family $(M_i)_{i \in I}$ of inner models together with a family of maps $(f_j)_{j \in J}$ between them.
- You may naturally regard them as a category $\mathcal{C}$. Add the identity for each M_i, and require that for each $f_j : M_1 \to M_2$, $f_k : M_2 \to M_3$ the composition $f_j \circ f_k$ exists; and that composition is associative.

§4. Why this manoeuvre?

- We know that maps of surroundings are not uniform. Cognitively speaking, there is no single internal model. (Check your atlas at home ...)
- The complexity of representations can largely be left implicit.
- Abstraction makes objects mentally tractable. The cost is shifted to the realisation functions.

Remark: Variability in meaning can (partially) be shifted to the realisation function (Andrei Rodin: *Axiomatic Method and Category Theory*. Synthese, 2014)

§5. A Case in Point: Spatial Relations

"left" denotes a relation between points in space ($\cong$ a subset of $\mathbb{R}^6$). Simplifying, $P = (p_1, p_2, p_3)$ is to the left of $Q = (q_1, q_2, q_3)$ iff $p_2 < q_2$. (The actual formula involves deviation from the axis and distance relative to the sizes of the objects etc.)

- But how to apply this?
- Where do the coordinates come from?

§6. The Egocentric View

Start with a Euclidean space E of points (the real space) as M_0. Take a point $o \in E$ (the observer). Associate to $p \in E$ the geometrical vector $\vec{op}$. This is the first model, M_1, a pointed space. The realisation map applies v to o to obtain p.

Use abstraction (of point pairs as vectors) to obtain a metrical vector space, M_2.

§7. The Virtual Observer

Now select three cardinal points, e_1, e_2, e_3 on the unit ball to represent the front, right and above direction. This establishes a coordinate system and allows to code each point p as a triple of numbers $(\gamma_1, \gamma_2, \gamma_2)$ such that

$$\vec{op} = \gamma_1 \cdot \vec{op}_1 + \gamma_2 \cdot \vec{op}_2 + \gamma_3 \cdot \vec{op}_3$$

We obtain the space $\mathbb{R}^3$ as M_3.

§8. The meaning of "left"

- The meaning of the word "left" is simply $(0, -1, 0)$ in M_3.
- This translates into a set of vectors in M_2, a set of points in M_1; finally, it translates into a set of real space points in M_0.

§9. Is it really that simple?

- Of course, you need to extend that to regions (I omitted that step).
- The placement of the observer depends on the word itself ("north" is different from "left" in that the directions depend only on the origin, o).
- Deviations of optimal directions are allowed, but there is a penalty (see eg the model by O'Keefe).
- The directions cut the circle into sections. Each section ends halfway into the other (north extends between northeast and northwest). Thus, you need to know the *system* of directionals to establish the exact boundaries.

Most of the complications are not language specific. They are learned outside of language. An exception is provided by the observer placement. This however is left implicit, i.e. is not part of model theoretic meaning.

§10. Aspects

Previously, I have called *aspect* M the realisation function for M. To fix ideas, assume a category $\mathcal{C}$ of (first order) models. A map $f_{ij} : M_i \to M_j$ is called a *recoding*. Let w be a word. Its meaning may be either a family of meanings m_i for each model M_i in the category. We require such a family to be coherent: $f_{ij}(m_i) = m_j$ for all i, j. Or it may be a single such element m_i. Then we put $m_j = f_{ij}(m_j)$. (Meaning percolates through $\mathcal{C}$).

> Coherent families can exist only if for each M_i, M_j only one recoding exists from M_i to M_j ($\mathcal{C}$ is "skeletal").

§11. Part 2. Transcendental logic

Theodora Achourioti & Michiel van Lambalgen: *A Formalization of Kant's Transcendental Logic*, The Review of Symbolic Logic, 2011 provide a formalisation of Kant's transcendental logic. The leading idea is that there is no unique representation and—initially—no object as such. There is a manifold of representations that need to be united in order to be "objective", that is, to be about an object.

> By synthesis in the most general sense ... I understand the action of putting different representations together with each other and comprehending their manifoldness in one cognition. (Kant: *Critique of Pure Reason*. A77/B103)

§12. Inverse Systems

A *directed set* is a pair $(S, \leq)$ such that $\leq$ is a partial order and for every $x, y \in S$ there is $z \in S$ such that $x, y \leq z$.

An *inverse system* $\mathfrak{I}$ over $(S, \leq)$ is a family $(M_i)_{i \in T}$ together with a family $(f_{ij})_{i \geq j} : M_i \to M_j$ of maps such that $f_{ij} \circ f_{jk} = f_{ik}$. ("Covariant functor from S into the category of structures".)

The *inverse limit* M of $\mathfrak{I}$ is formed by all the functions $\lambda : \prod_{i \in S} M_i$ such that $\lambda(j) = f_{ij}(\lambda(i))$, for all $i \leq j$.

§13. Appearances and objects

> Appearances are not things in themselves, but themselves only representations, which in turn have their object, which therefore cannot be further intuited by us, and that may therefore be named the non-empirical, i.e., the transcendental object = X. The pure concept of this transcendental object (which in reality throughout all our cognition is always one and the same = X), is what can alone confer upon all our empirical concepts in general relation to an object, i.e., objective validity. This concept cannot contain any determinate intuition at all, and therefore concerns nothing but that unity which must be encountered in a manifold of cognition insofar as it stands in relation to an object. This relation, however, is nothing other than the necessary unity of self-consciousness, thus also of the synthesis of the manifold, through a common function of the mind for combining it in one representation. (A109)

§14. Return to meanings

- If objects can't be had before the synthesis, there is trouble for explaining meanings in terms of objects.
- If appearances are manifold, and judgments are based on appearances, maybe we can instead explain meanings in terms of appearances.
- If the family of appearances is coherent it can be synthesized. Moreover, meanings can be translated into other appearances using the transfer functions.

§15. The (dis)unity of the two approaches

- While the issue of recoding goes between any two models (in principle), the inverse system admits arrows only in one direction.
- We can generalise the theory to arbitrary categories and limits (but we may lose some of the logical appeal).
- Recoding allows to simplify the representations, showing only what is essential. Meanings can be entered where the representation is most simple. Whether or not that meaning percolates though the system is left implicit.

§16. Thank you!