Degree and Quantity: Semantics and Conceptual Representation

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Referential Semantics One Step Further

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The semantics of degree

- Reference to / comparison of degrees:

  Anna is 1,65 m tall. Zoe isn't that tall.

  Anna is taller than Zoe.

- The degree semantic framework:
  - Enrich ontology to include degrees (type *d*)
  - Degrees organized into scales $S = \langle D, >, DIM \rangle$
    - *D* a set of degrees
    - $>$ an ordering relation on *D*
    - *DIM* a dimension of measurement

(Bartsch & Venemann 1973; Cresswell 1977; Bierwisch 1989; Kennedy 1997; Heim 2000; among many others)

Degree-semantic framework

- Broad, flexible application
  - Gradable adjectives, quantity expressions, verbs, …
  - Degree modification, comparison; telicity, …
- But fundamental questions remain open
  - What sort of things are degrees?
  - What is the structure of the domain D_d?
- **Main thesis:** The degree-semantic framework can be enriched and strengthened by incorporating findings on the mental representation of quantity and degree

1. Ordering strength

- Basic definition of scale imposes no restrictions on $\succ$.
- Cresswell 1977: Only weak assumptions:
  - "…tempting to think of $\succ$ as at least a partial ordering"
    - transitive
    - antisymmetric
  - Unimportant whether strict or not, total or not
  - Maybe we shouldn't even insist on transitivity/ antisymmetry

1. Ordering strength

- Recently: $>$ has property of **totality**

  For any distinct d, d', either d $>$ d' or d' $>$ d

  - Kennedy 2007: "A set of degrees totally ordered with respect to some dimension constitutes a scale"
    - Also: Moltmann 2009; Beck 2011; Wellwood 2014; among many others
  - Related view (Krifka 1989; Rothstein 2010): Degrees as real numbers ordered by ≥
    - An exception: Lassiter (to appear) on modality

Orderings in cognition

Characterized by **tolerance** rather than total ordering.

- Psychophysics: Discriminability of two stimuli (e.g. weight of objects, loudness of tones, brightness of lights) subject to ratio-dependent threshold, the 'just noticeable difference' JND (Gescheider 2015)
- Preference: Lack of preference between two options may be intransitive (Luce 1956)
  - Chocolate chip cookie problem

- Quantity comparison: In tasks that preclude precise counting, performance characterized by size and distance effects that can be described by Weber's law (Dehaene 1997; Feigenson et al. 2004; a.o.)

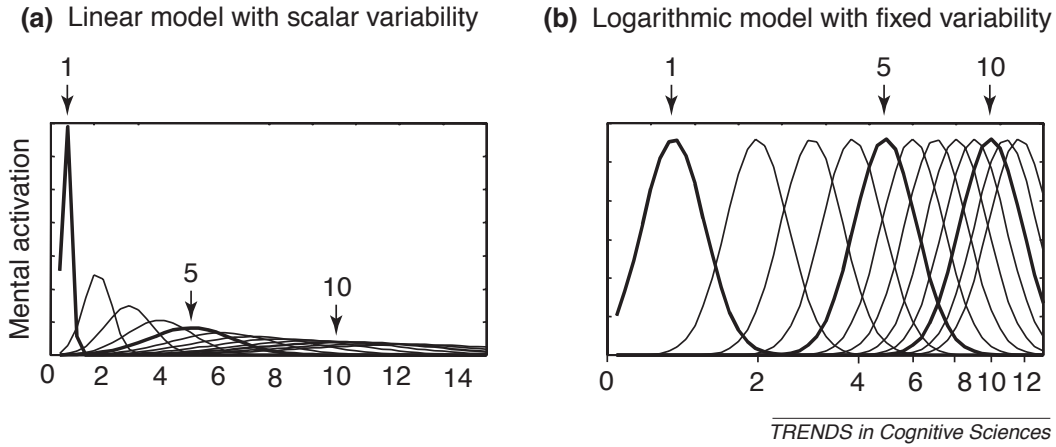
Number Cognition

Approximate Number System: Non-species-specific capacity to represent and manipulate approximate quantity

- Numerosities represented as patterns of activation on continuous mental number line

  'mental magnitudes with scalar variability'

- Modeled as Gaussians whose widths increase in proportion to their magnitude



Feigenson et al. 2004

Degree semantics with tolerant scale structures

What would happen extend the degree-semantic framework to allow scales based on models of the ANS?

Tolerant ordering: $d_1 \sim d_2$ and $d_2 \sim d_3$ but $d_1 > d_3$

- 'Significantly greater than' comparisons (Fults 2009; Solt 2016)

  Anna is tall compared to Zoe.

  $$\mu_{HEIGHT}(Anna) >_{tolerant} \mu_{HEIGHT}(Zoe)$$

  Most of the marbles are blue.

  $$\mu_{\#}(\text{blue marbles}) >_{tolerant} \mu_{\#}(\text{non-blue marbles})$$

- Approximate numerical constructions?

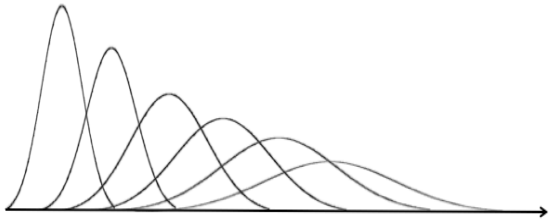
  (about 50) linguists

Analogue magnitude scale

Degrees as…

… intervals?

… probability distributions over precise points?



Ordering relation $>$ as …

… semiorder (Luce 1956)?

… probabilistic function?

- Ratio dependence problematic to axiomatize

2. Dimensions without units

- Potential objection: It is plausible to assume degrees/ scales as part of the ontology for dimensions such as cardinality and height with corresponding measurement units. But what about non-measureable dimensions such as beauty?

> "Must we assume the kalon as a degree of beauty or the andron as a degree of manliness? Degrees of beauty may be all right for the purposes of illustration but may seem objectionable in hard-core metaphysics" (Creswell 1977, p. 281)

Degrees as equivalence classes

(Cresswell 1977; Bale 2008, Lassiter 2011)

- Start with a weak order *R* on individuals

  E.g. *'is at least as tall as'* or *'is at least as beautiful as'*

- Define an equivalence relation ≈

  $a \approx b$ iff for all c: aRc iff bRc and cRa iff cRb

- Build equivalence classes

  $\bar{a}=\{x : x \approx a\}$ - these are degrees

- Define ordering relation $\succ$ on degrees/equivalence classes on the basis of *R*

  - This is an ordinal scale! (Stevens 1946)

Evidence from linguistics

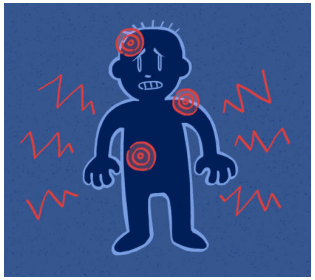
Speakers behave as if scales underlying non-measurable gradable expressions is stronger than ordinal level:

- Distance comparisons
  - Anna is much taller/older/heavier than Zoe.
  - Anna is much happier/more beautiful/more talented than Zoe.
- Ratio modifiers
  - Anna is twice as tall/old/heavy as Zoe.
  - ??Anna is twice as short/young/light as Zoe.
  - Anna is twice as happy/beautiful/talented as Zoe.
    - Sassoon 2009: *happy* etc., like *tall* etc., lexicalize ratio scales.
- But…
  - Anna is 3.1 times as tall/old/heavy as Zoe.
  - ??Anna is 3.1 times as happy/beautiful/talented as Zoe.

Evidence from cognition

Work in psychophysics and related fields has shown that a broad range of perceptions and attitudes can be measured at the **interval** or **ratio** level

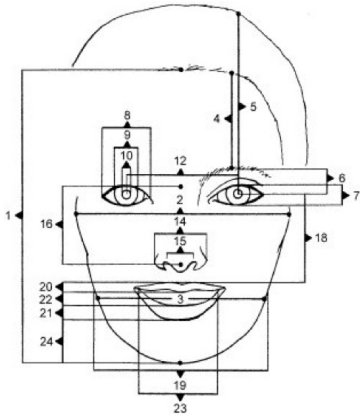
Perception: loudness, brightness, taste (salt, sugar), smell (e.g. coffee), pressure, temperature (Stevens 1957)



Pain (Price et al. 1983)

Unpleasantness of sounds (Ellermeier et al. 2004)

Scenic beauty (Daniel et al. 1977, Ribe 1988)



Facial attractiveness (Kissler & Bäuml 2000)

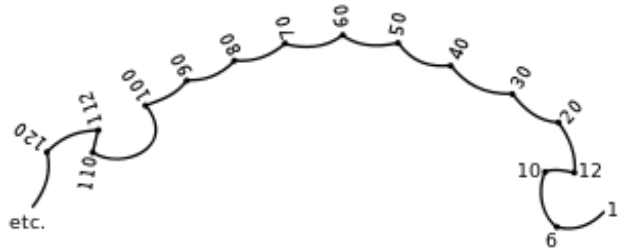
Conclusions on scale type

- Even for dimensions without standard units, an ordinal scale derived via the equivalence-class procedure is not consistent with
  - Performance on psychophysics tasks
  - Linguistic behavior
- Seem instead to require intermediate scale type:
  - Stronger than ordinal: distance between scale points meaningful
  - Weaker than true ratio: no standard units; no precise ratio comparisons
  - Perhaps approximate magnitude scale the right metaphor here as well

3. Spatial orientation

Close relation in cognition between **quantity and measure** and **space**:

- SNARC effect: spatial-numerical association of response codes (Dehaene et al. 1993)
  - Left-right orientation of mental number line
- Number forms – a form of synesthesia (Galton 1881)



- Across cultures, time conceptualized in terms of space (Núñez & Cooperrider 2013)
- Common structures in parietal cortex involved in representation of space, number, time and other magnitudes (Bueti & Walsh 2009)

Spatial metaphor

Using the language of space to talk about…

…number and measure

high ground / *high number* / *high price*

The dog is under the table / *The lamp hangs over the table*
John found over / under 50 typos in the manuscript
For children with body weight over 20 kg…

The temperature rose

…time

Jan stond voor zijn huis	'Jan stood in front of his house'
voor 11 uur	'before 11 o'clock'

Move the meeting forward / push the meeting back
The winter is fast approaching

Corver & Zwarts 2006; Núñez & Cooperrider 2013; Nouwen 2016; among many others

Some puzzling disconnects

- Prevalence of vertical metaphors – particularly for number
- Lack of left/right metaphors, in spite of…
  - Left-to-right orientation of mental number line (in Western culture)
  - Left-to-right conceptualization of temporal sequence (some cultures)
- Some cultures: spatial conceptualization of time without spatial metaphors

  ➢ Argues against equating mental representations and semantic scales (Nouwen 2016)

Scale structure and metaphor

- Nouwen 2016: Scale structure provides a clue to orientation of spatial metaphors
  - Scale of number is a ratio scale (Stevens 1946)
  - Only vertical axis has crucial property of ratio scale, namely fixed 0 point (the ground)

    The scalar metaphor condition: expressions that function on a scale S can only be metaphorically used on a scale S' if S is at least as high a level of measurement as S', where the relevant hierarchy of levels is: ordinal < interval < ratio.

    ➢ Correctly predicts possibility of horizontal metaphors for interval/ordinal scales, particularly clock time (though not temperature)

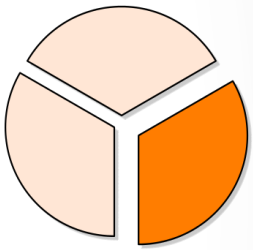
Approximation and visualization

Number/measure often communicated approximately:

It's a quarter after four.

- Speaker's watch reads 4:17

A third of Americans (34%) read the bible daily.



- Rounding is common (van der Henst et al. 2002)
- Rounded values easier to process (Solt et al. 2016)

  - Preference for values that can be easily visualized?

Conclusions

- Degree-semantic framework can be enriched by view from cognition
  - Scale structure / nature of degrees
  - Metaphorical language
  - Expression choice
- Formalizing such insights is far from straightforward

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