day 4

ESSLLI 2016
Bolzano, Italy

Logical foundations of databases

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Recap

- **Conjunctive Queries** (correspondence with SQL and Relational Algebra)
- **Homomorphisms and canonical structure**
- **Evaluation of CQ** (NP-completeness)
- **Containment, Equivalence, Minimisation of CQ** (NP-completeness)
- **Extension to functional dependencies** (chased canonical structure)
- Acyclic Conjunctive Queries

Acyclic CQ's : Definition

On graphs: CQ ϕ is **acyclic** if G_ϕ is tree-like

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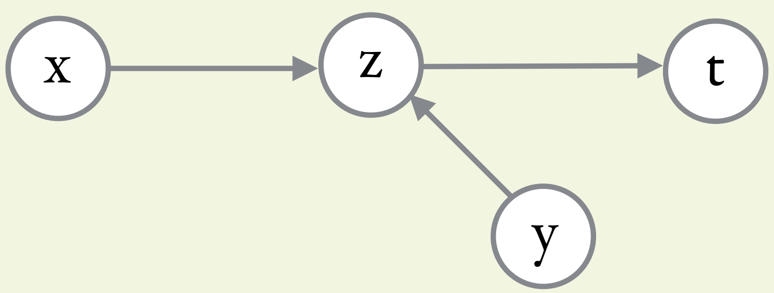
$\phi(x,y) = \exists z \,.\, E(x,z) \wedge E(z,t) \wedge E(y,z)$



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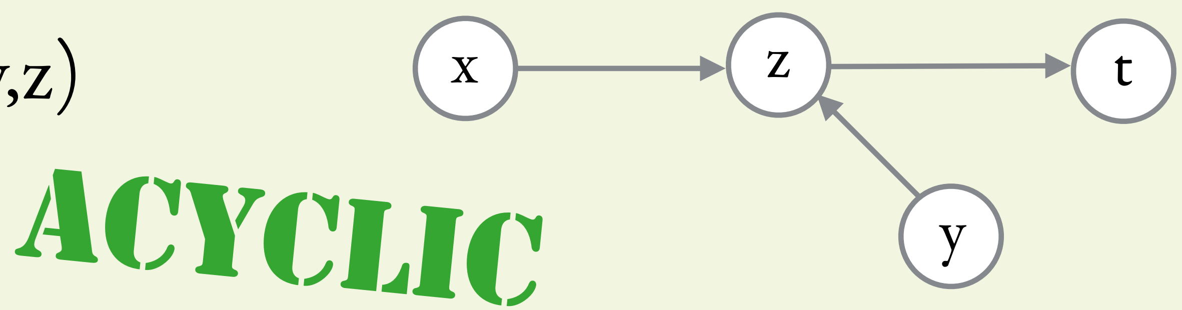

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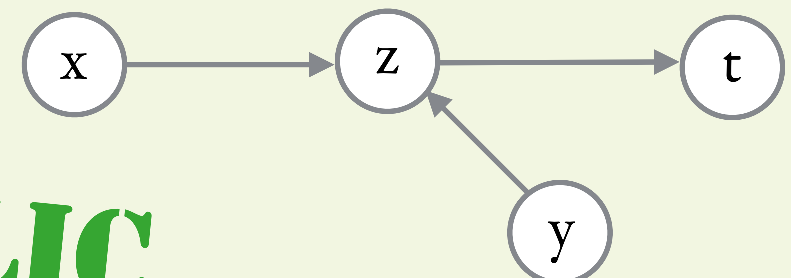

ACYCLIC

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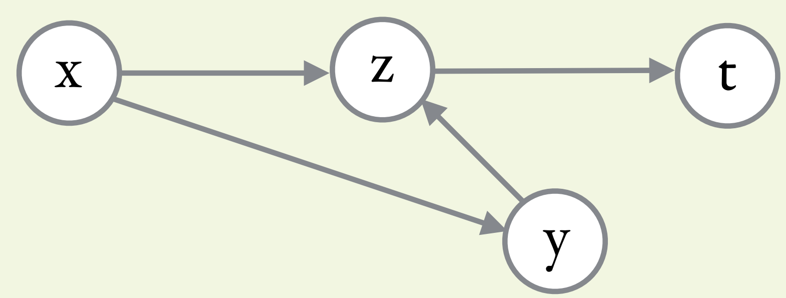

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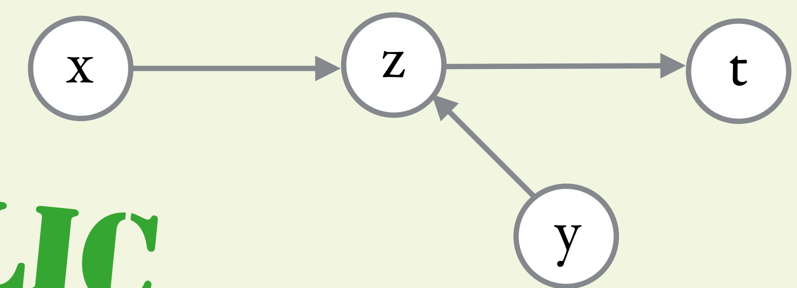

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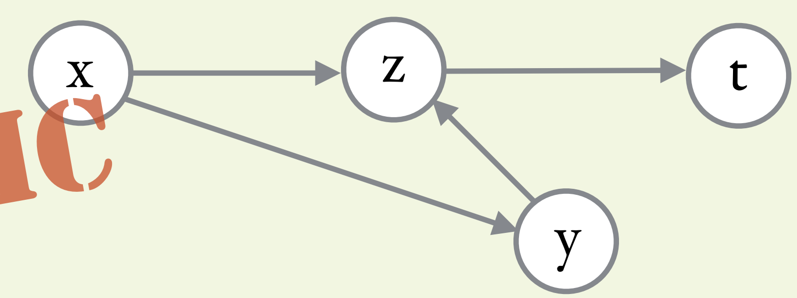

NON ACYCLIC

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On general structures: a CQ ϕ is **acyclic** if it has a join tree

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A **join tree** is a tree T st:

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- for every variable x of ϕ the set of $R_i(\bar{z}_i)$'s with $x \in \bar{z}_i$ forms a subtree of T

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Alternatively, if its canonical hyper-graph is α-acyclic.

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Acyclic CQ's

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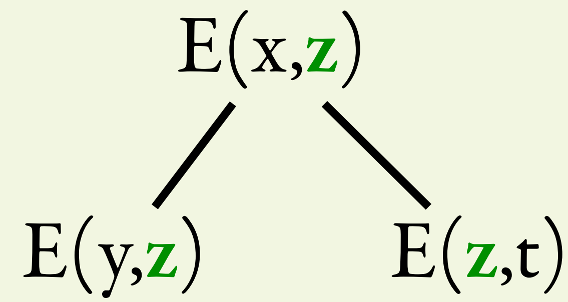
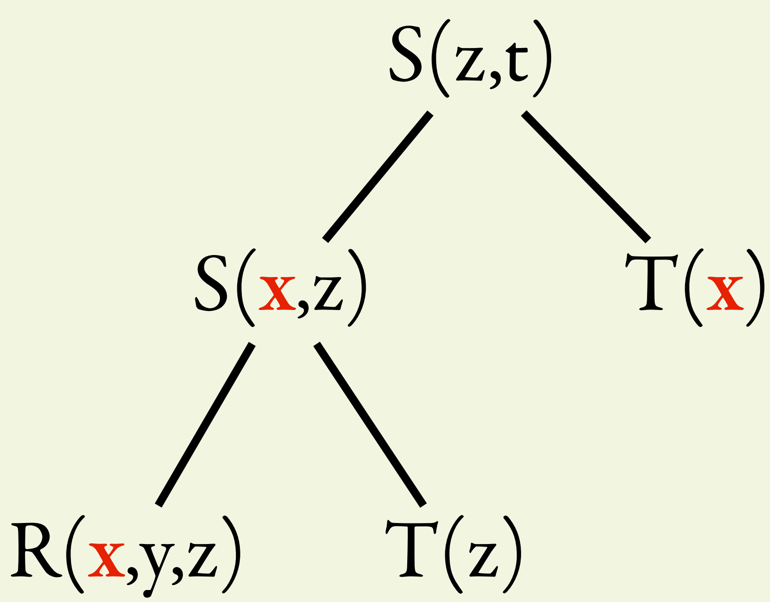
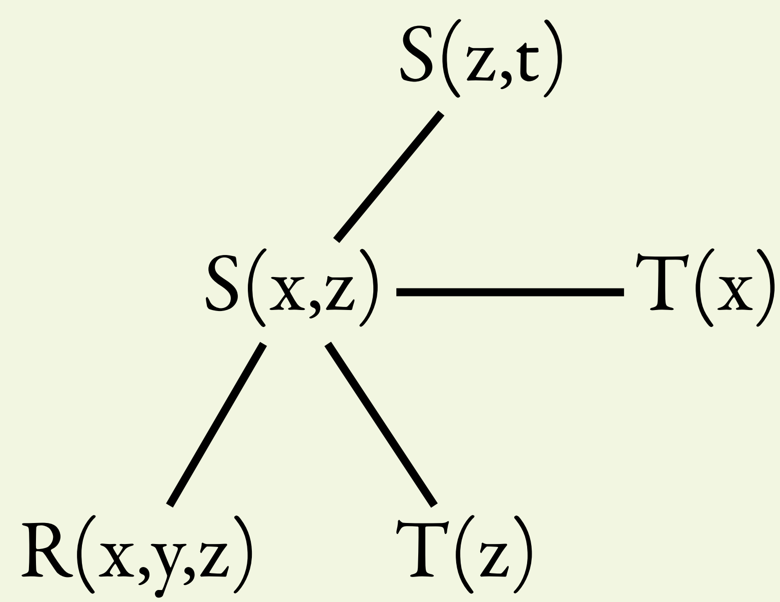


join tree

$\phi = \exists x,y,z,t\,.\, R(x,y,z) \wedge S(z,t) \wedge S(x,z) \wedge T(z) \wedge T(x)$



not a join tree



a join tree

Acyclic CQ's

$\phi_1 = R(x_1,x_2,x_3) \wedge R(x_1,x_6,x_5) \wedge R(x_3,x_4,x_5)$ join tree?

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$\phi_2 = R(x_1,x_2,x_3) \wedge R(x_1,x_6,x_5) \wedge R(x_3,x_4,x_5) \wedge R(x_1,x_3,x_5)$ join tree?

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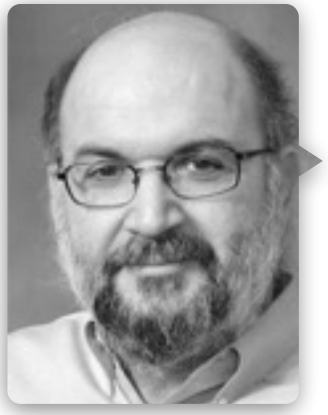
$$\begin{tikzcd}[arrows=dash]
 & R(x_3,x_4,x_5) \arrow[d] & \\
 & R(x_1,x_3,x_5) \arrow[dl] \arrow[dr] & \\
R(x_1,x_2,x_3) & & R(x_1,x_6,x_5)
\end{tikzcd}$$

Acyclic CQ's



The **evaluation problem** for acyclic CQ sentences is in $O(|\phi|.|D|)$

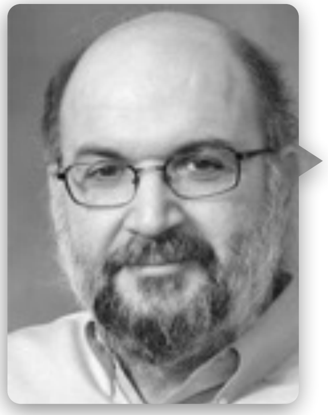
[Yannakakis]

The **semi-join**

$$R \ltimes_{\{i_1=j_1,\ldots,i_n=j_n\}} S = \{ (x_1,\ldots,x_n) \in R \mid \text{there is } (y_1,\ldots,y_m) \in S \text{ where } x_{i_k} = y_{j_k} \text{ for all } k\}$$

Note: $R \ltimes_{\{i_1=j_1,\ldots,i_n=j_n\}} S \subseteq R$

Acyclic CQ's



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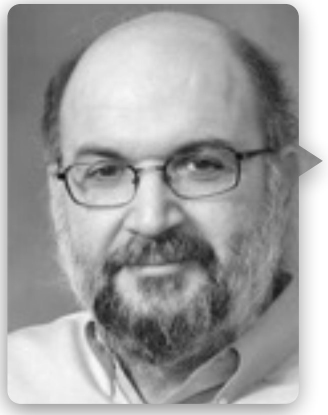
[Yannakakis]

1. Compute the join tree T for ϕ
2. Populate the nodes of T with corresponding relations of D
3. For every leaf $S(x_1,\ldots,x_n)$ with parent $R(y_1,\ldots,y_m)$ perform
   $$R \ltimes_{\{i=j \mid x_i = y_j\}} S$$
   and delete the leaf $S(x_1,\ldots,x_n)$.
4. Repeat until we are left with one node. If it contains a non-empty relation, then D satisfies ϕ, otherwise it does not.

Acyclic CQ's



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[Yannakakis]

in linear time

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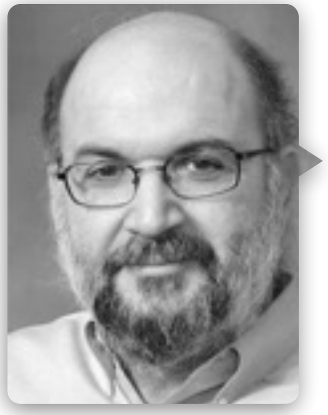

The evaluation problem for acyclic CQ sentences is in O(|ϕ|.|D|)

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remove all the tuples from the parent that do not match a tuple from the child

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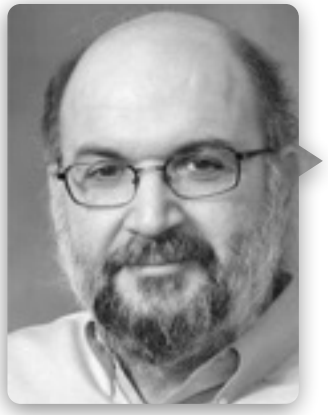

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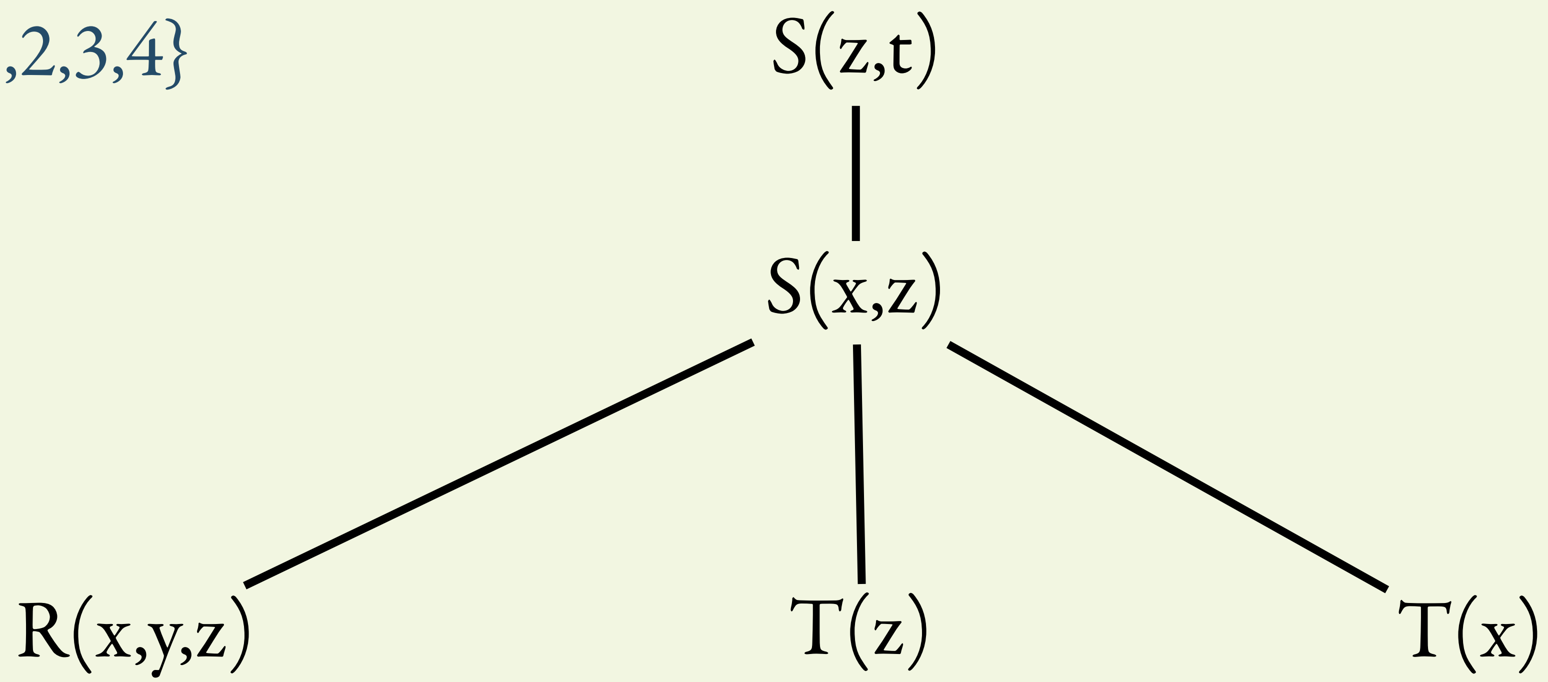
Acyclic CQ's

(combined c.)

The **evaluation problem** for acyclic CQ sentences is in $O(|\phi|.|D|)$

$\phi = \exists x,y,z,t \,.\, R(x,y,z) \wedge S(z,t) \wedge S(x,z) \wedge T(z) \wedge T(x)$

R={(1,4,4),(4,1,4)}
S={(4,5),(5,2),(4,4)}
T={1,2,3,4}



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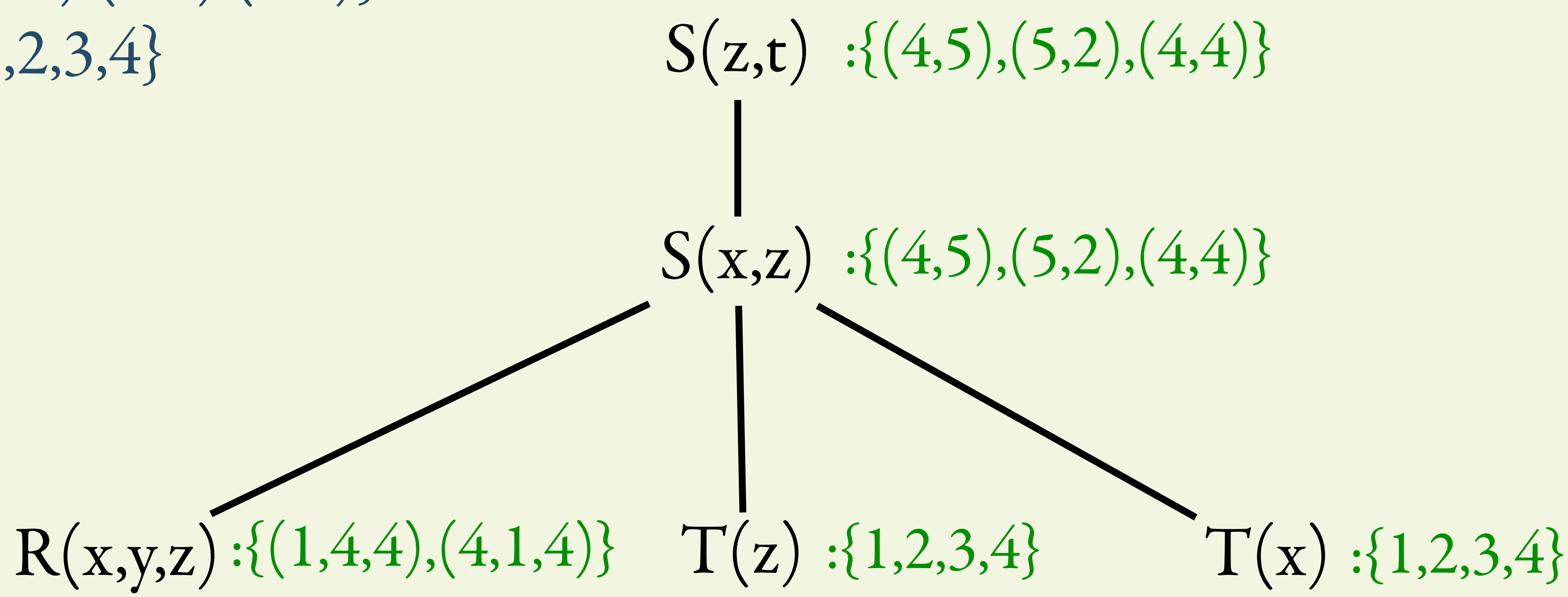

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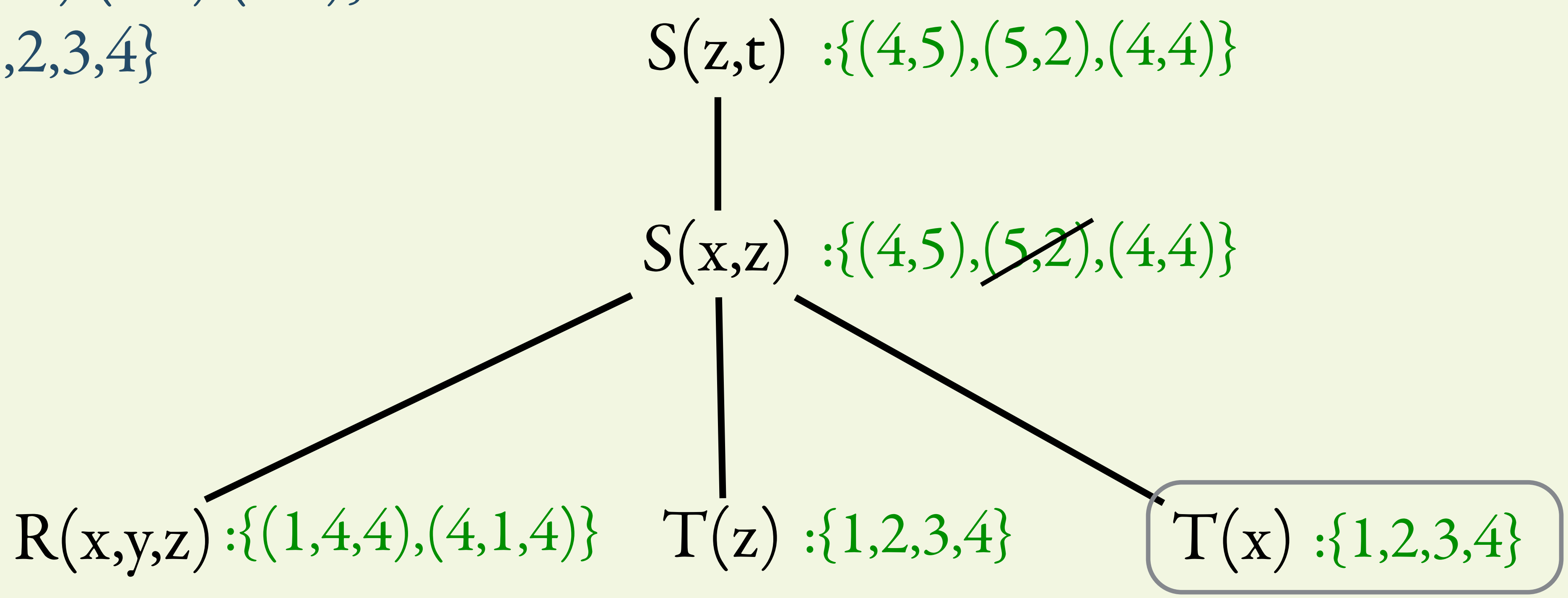

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S(z,t) :{(4,5),(5,2),(4,4)}

S(x,z) :{~~(4,5)~~,~~(5,2)~~,(4,4)}

R(x,y,z) :{(1,4,4),(4,1,4)} T(z) :{1,2,3,4} T(x) :{1,2,3,4}

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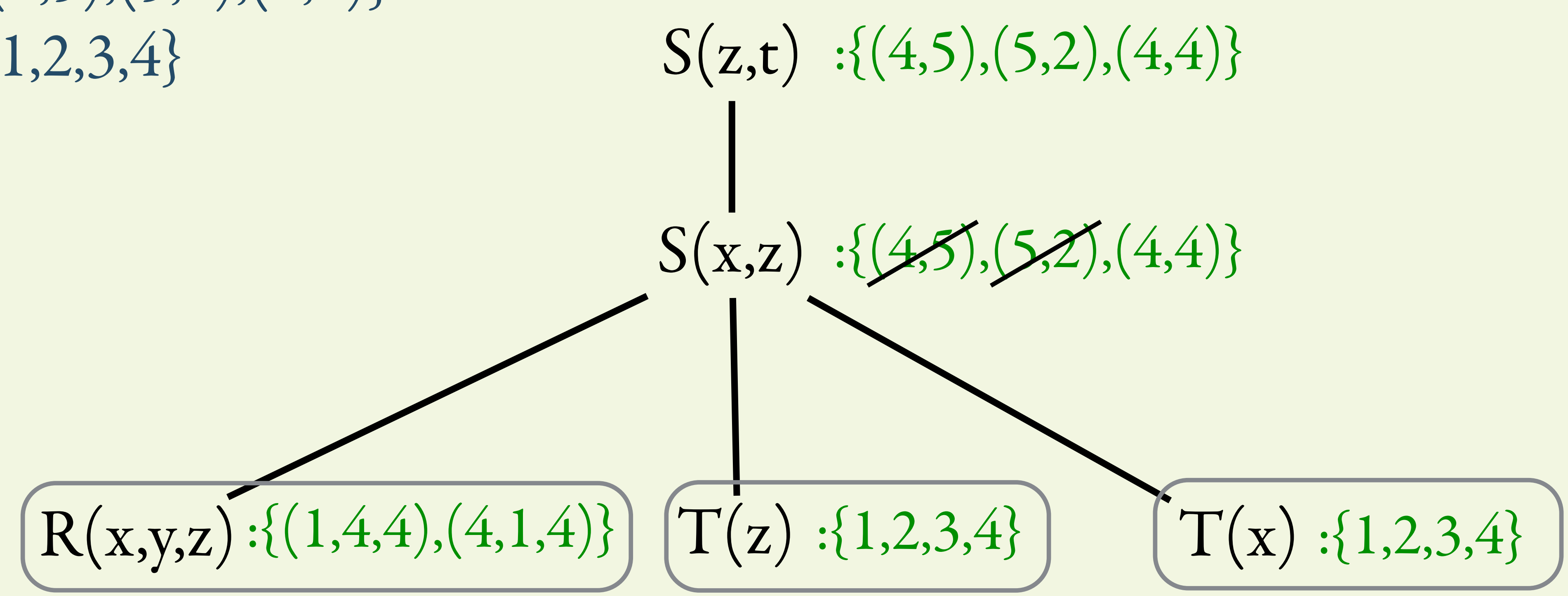

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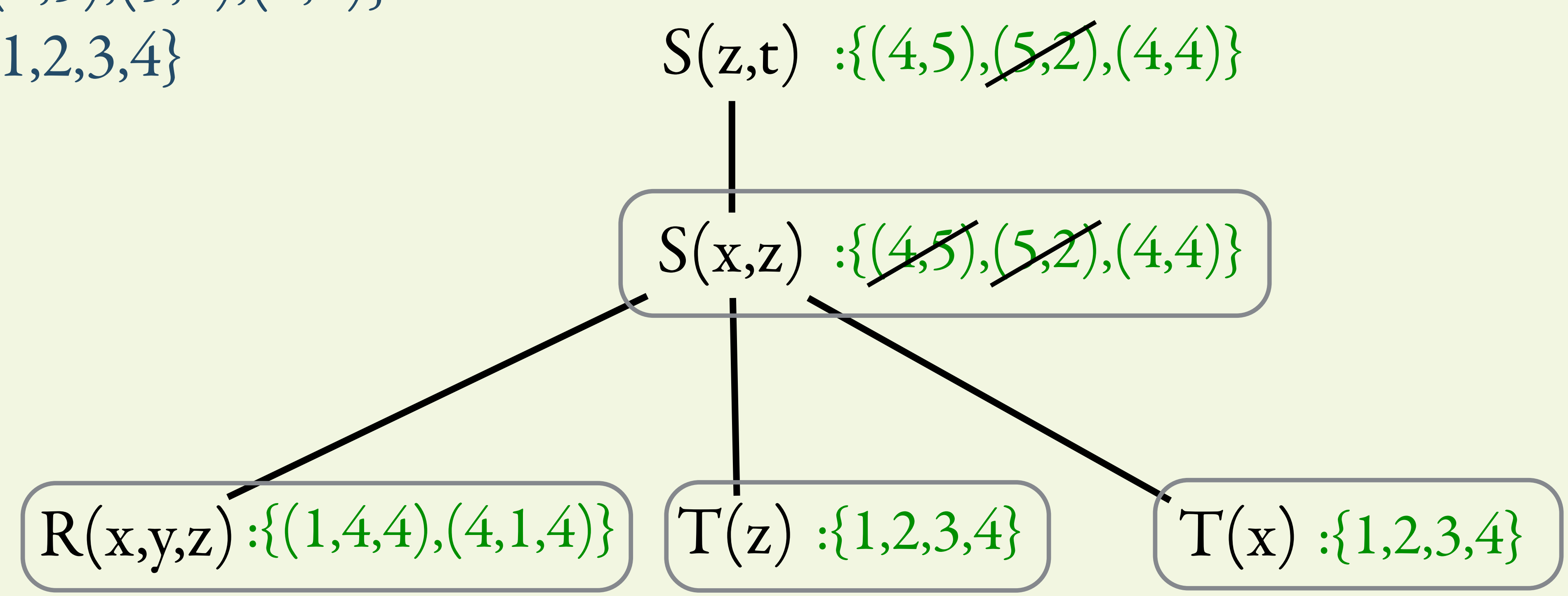

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$S(z,t)$ $:\{(4,5),\cancel{(5,2)},(4,4)\} \neq \emptyset$

$S(x,z)$ $:\{\cancel{(4,5)},\cancel{(5,2)},(4,4)\}$

$R(x,y,z)$ $:\{(1,4,4),(4,1,4)\}$ $T(z)$ $:\{1,2,3,4\}$ $T(x)$ $:\{1,2,3,4\}$

Acyclic CQ's

How to compute a join tree?

GYO reducts [Graham, Yu, Ozsoyoglu]

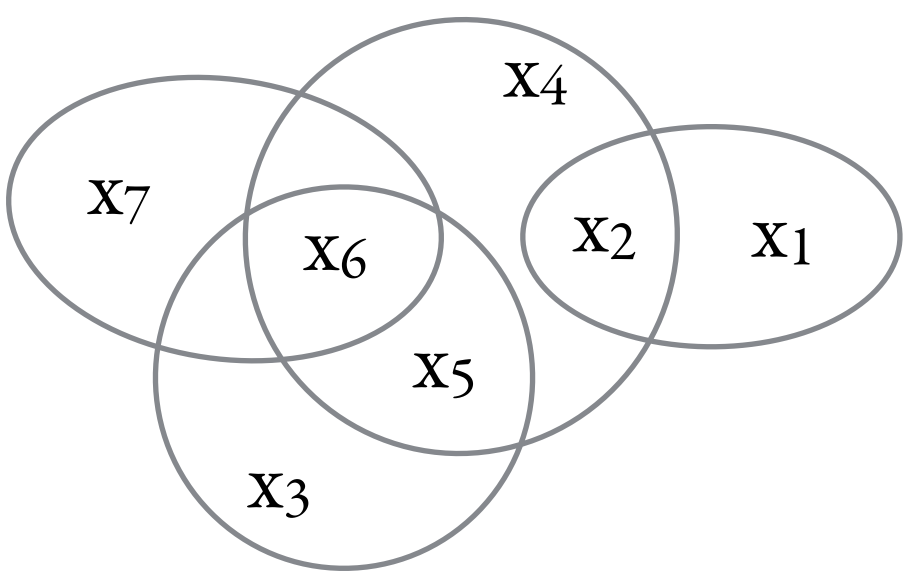
An **ear** of a hypergraph (V,E) is a hyperedge **e** in E such that one of the following conditions holds:

(1) There is a **witness e'** in E, such that e' ≠ **e** and each vertex from **e** is either

(a) **only** in **e** or

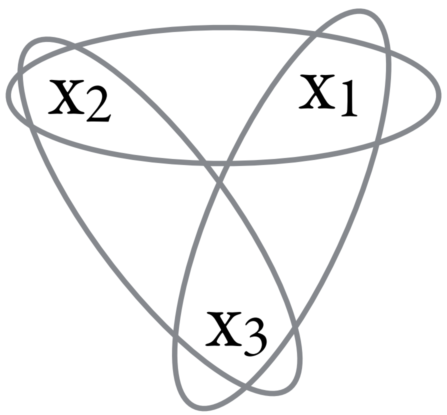
(b) in e'; or

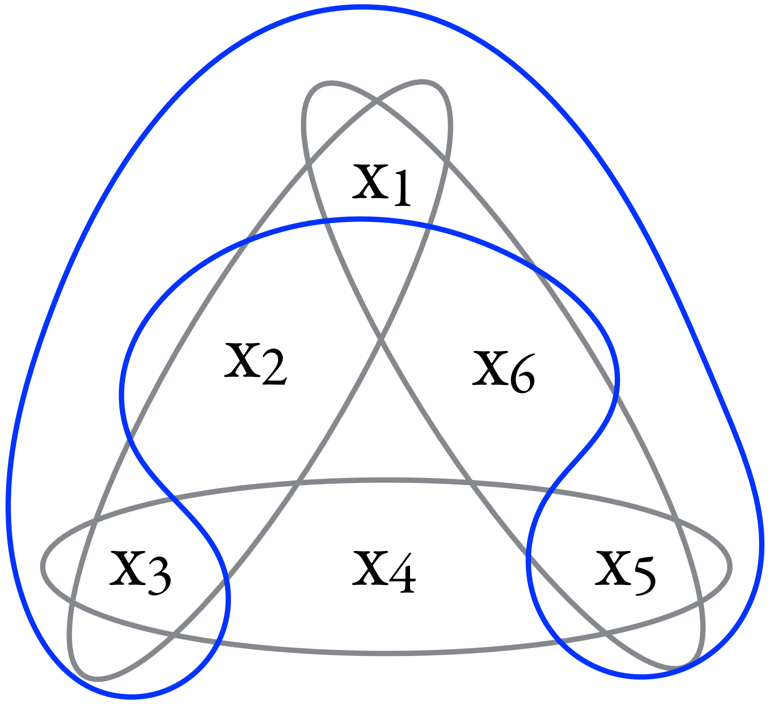
(2) **e** has no intersection with any other hyperedge.



ears?

Ears?





Ears!

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Theorem: TFAE

- The GYO **reduct** of a hyper graph **G** is empty
- A CQ ϕ having **G** as underlying canonical hyper-graph is acyclic
- The hyper graph **G** is α-acyclic

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We can test acyclicity by computing the GYO reduct!

Acyclic CQ's

How to compute a join tree?

GYO algorithm [Graham, Yu, Ozsoyoglu]

Given the query $\phi = R_1(X_1) \wedge \cdots \wedge R_n(X_n)$
Consider its canonical structure G_ϕ
  For $R_i(X_i)$ an ear with witness $R_j(Y_j)$
  Put an edge between $R_i(X_i)$ and $R_j(X_j)$, and remove R_i from ϕ.
  Repeat.

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E.g.
R(x,y,z), S(x,y), T(x,x), R(x,x,y), T(y,y)

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T(x,x)

R(x,y,z) R(x,x,y) T(y,y)

S(x,y)

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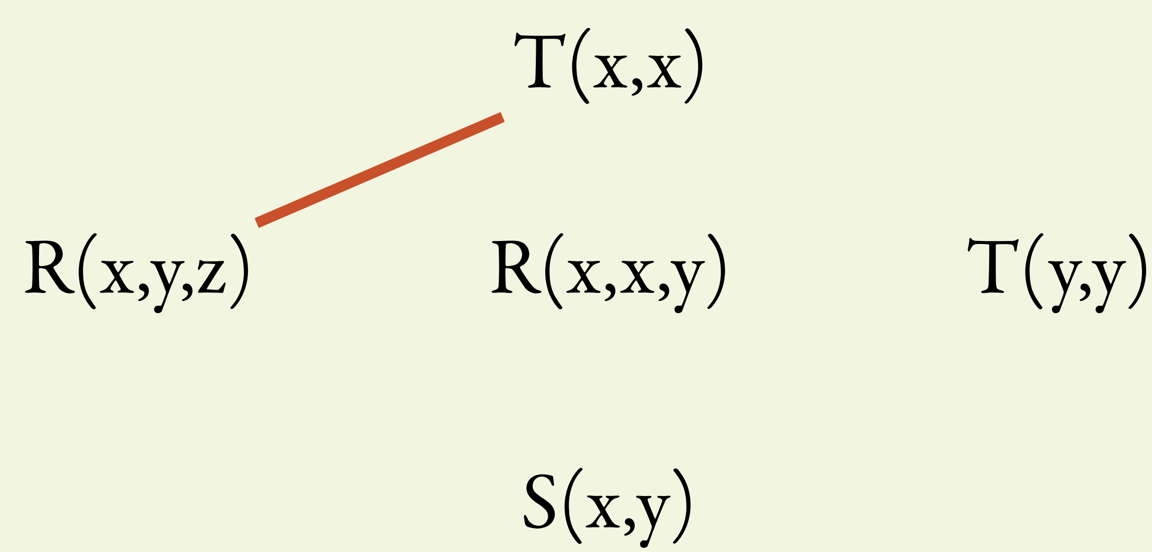

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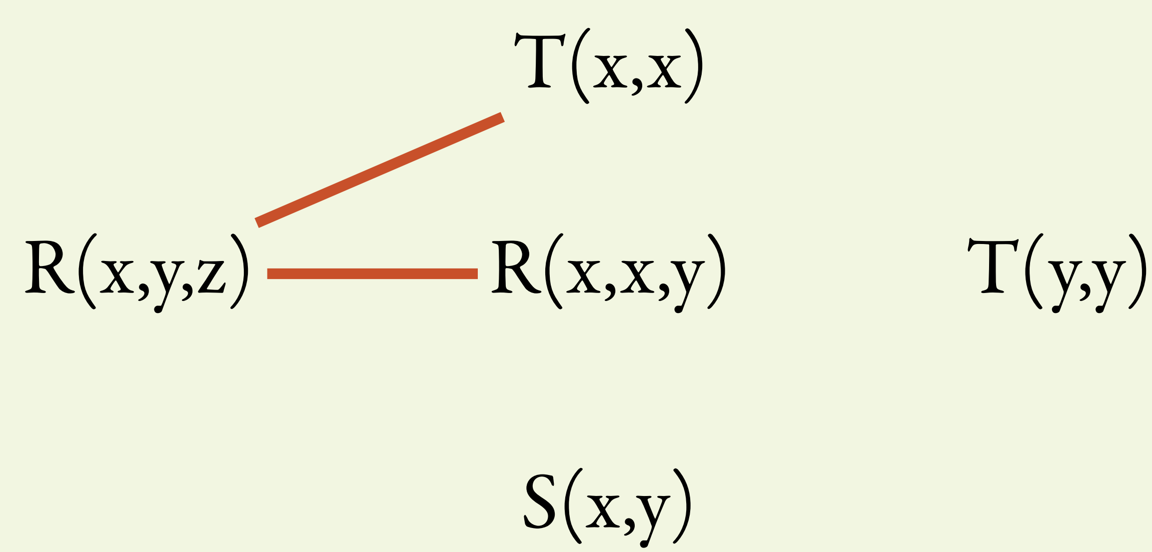
Repeat.

E.g.
R(x,y,z), S(x,y), ~~T(x,x)~~, ~~R(x,x,y)~~, T(y,y)



Acyclic CQ's

How to compute a join tree?

GYO algorithm [Graham, Yu, Ozsoyoglu]

Given the query $\phi = R_1(X_1) \wedge \cdots \wedge R_n(X_n)$
Consider its canonical structure G_ϕ
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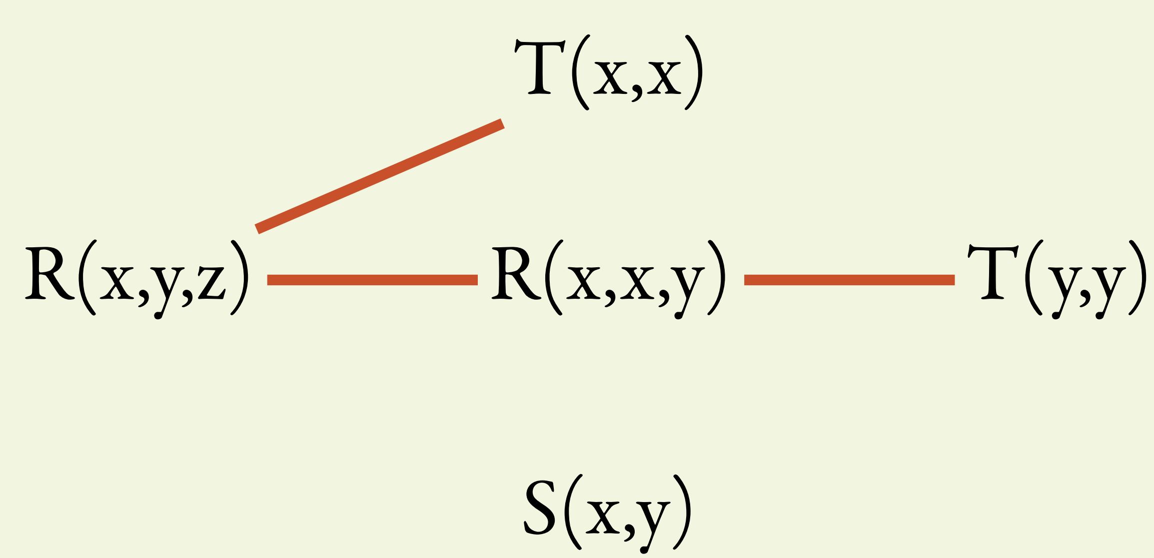

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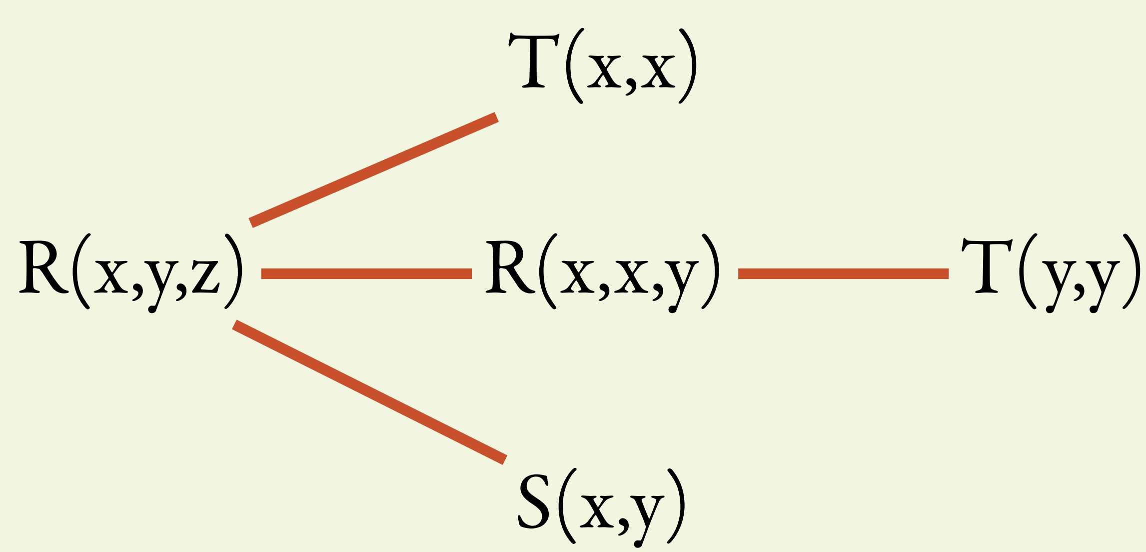

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Remove ears until you're left with only one!

E.g.
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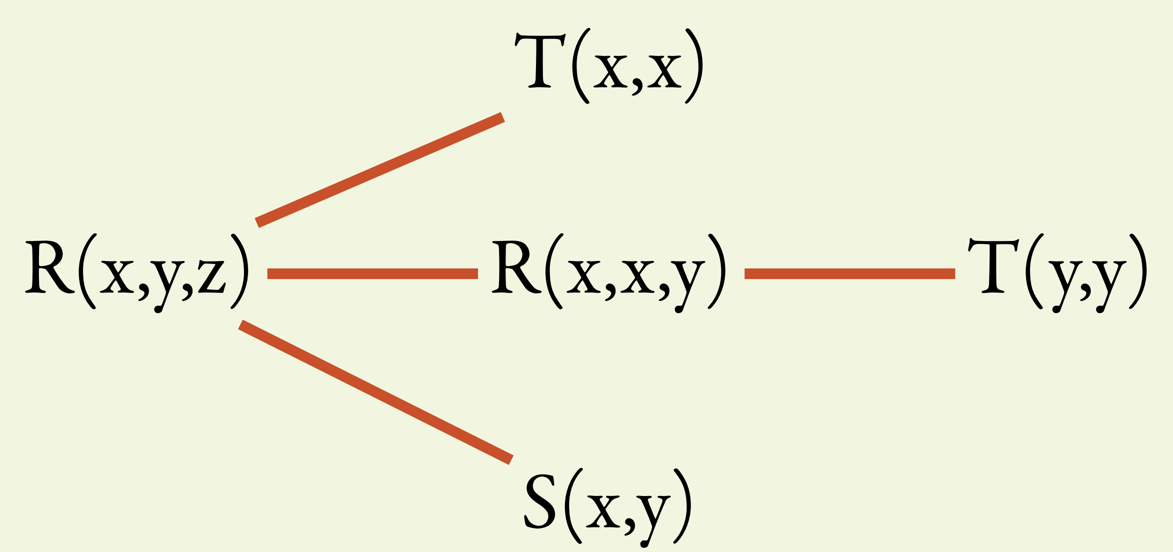

Acyclic CQ's

[Gottlob, Leone, Scarcello]

- Evaluation problem for boolean ACQ's is LOGCFL-complete

- NL $\subseteq$ **LOGCFL** $\subseteq$ AC^1 $\subseteq$ NC^2 $\subseteq$ P

the class of problems
logspace-reducible to
a context-free language

Beyond acyclic CQ's

Treewidth = a measure of the cyclicity of (hyper-)graphs

$$\mathrm{tw} : \mathrm{CQ} \longrightarrow \mathbf{N}$$

For a fixed k,

the evaluation pb for queries of tw $\leq k$

can be done in **polynomial time**.

[Chekuri, Rajaraman]

Idea: the lower tw(ϕ), the more ϕ resembles a tree

Tree-width, definition

A **tree decomposition** of a graph G:

A bunch of graphs with a special edge "· · · ·" between their nodes so that

1) they have a **tree shape** and

2) collapsing "· · · ·" edges $\rightsquigarrow$ G

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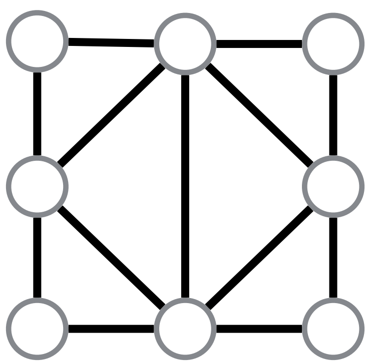

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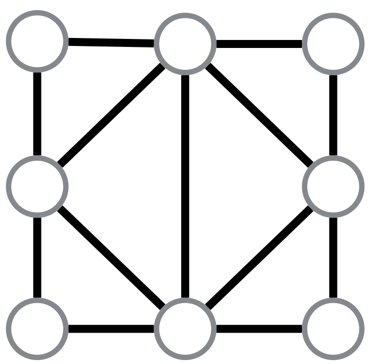

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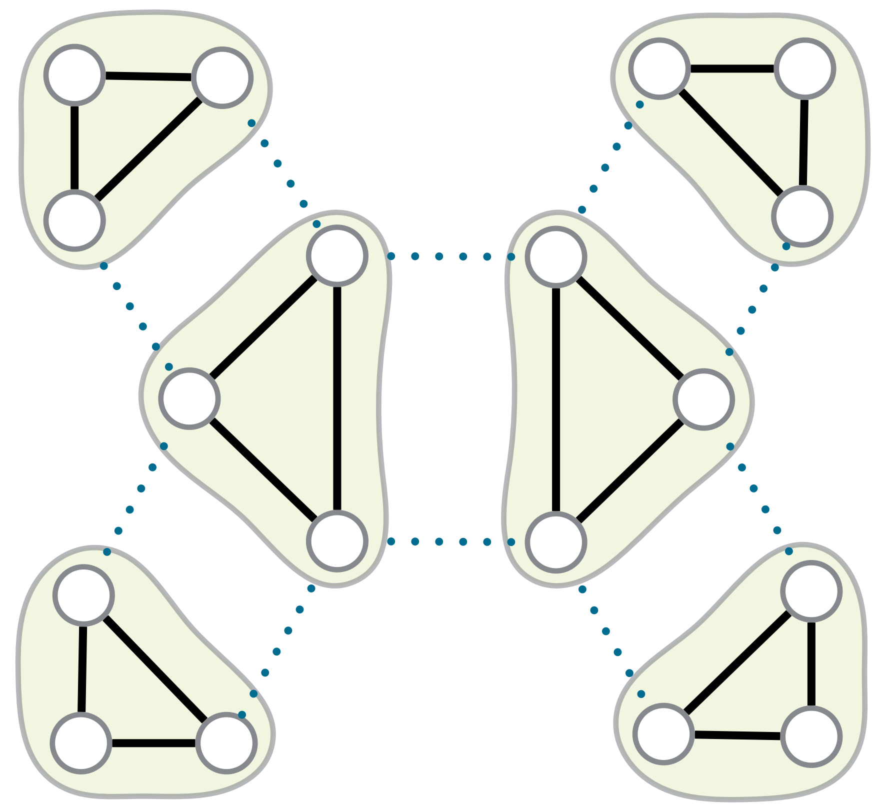

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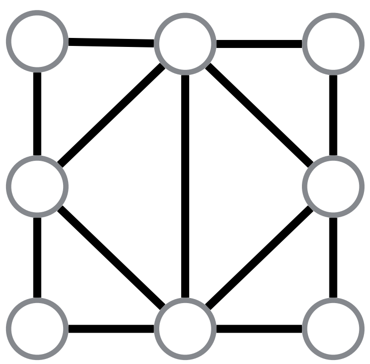

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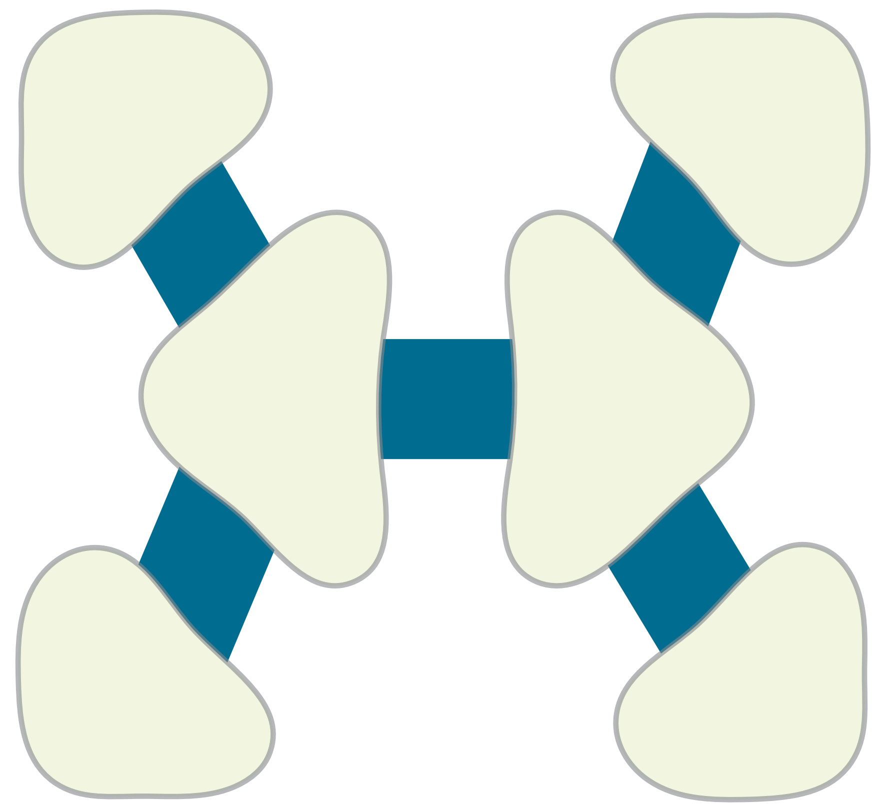

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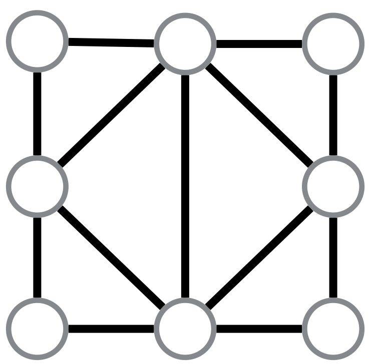

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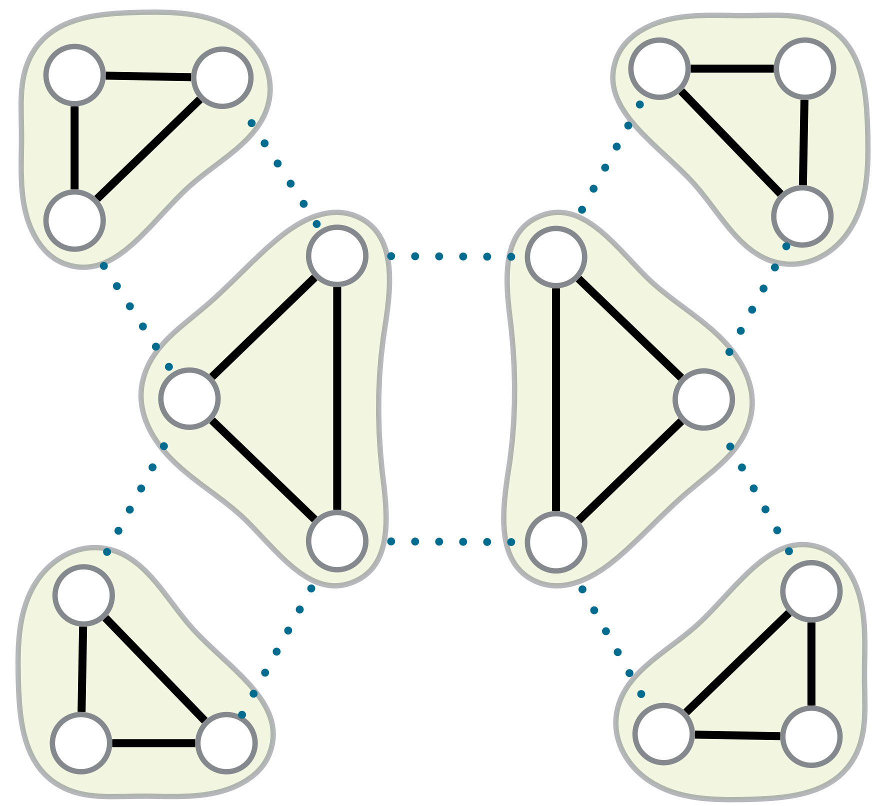

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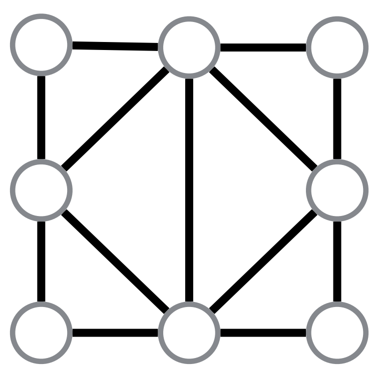

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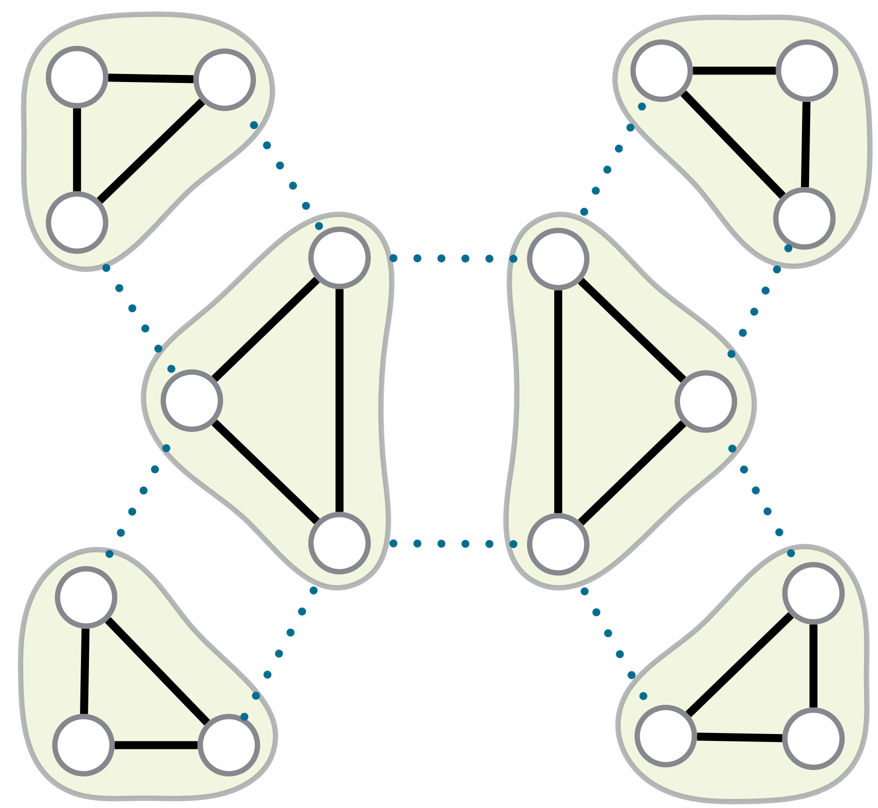

tree decomposition of G

width of decomposition = maximum size of graphs −1

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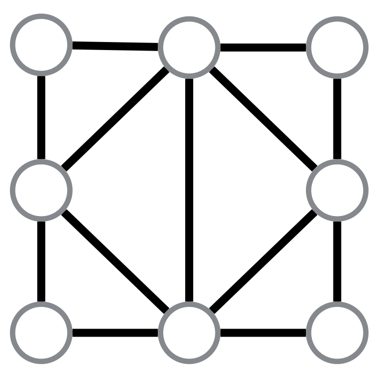

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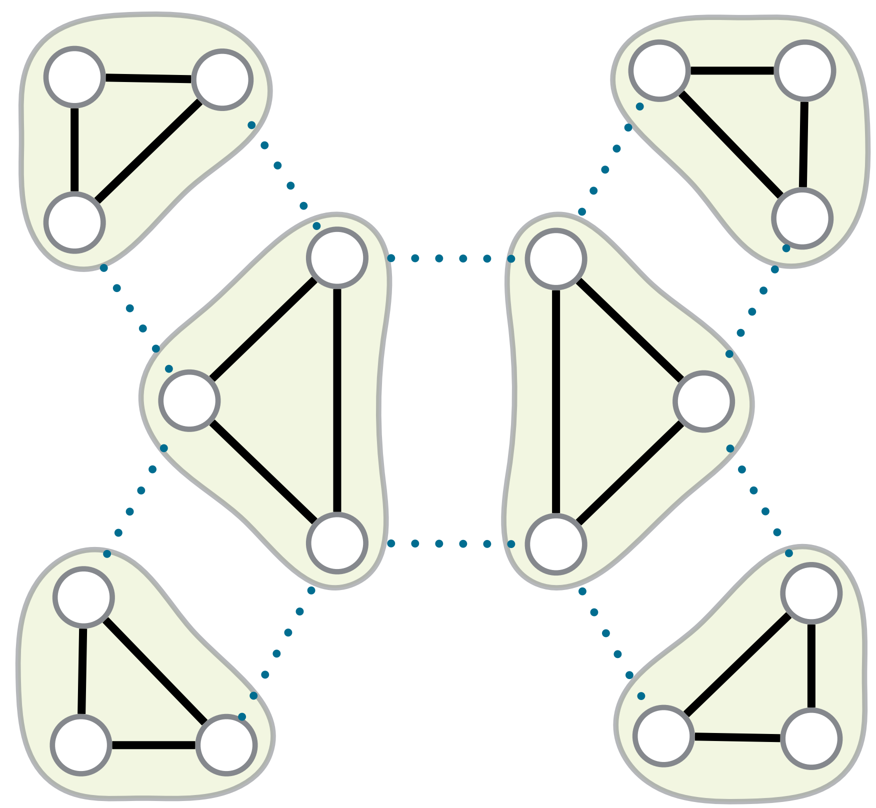


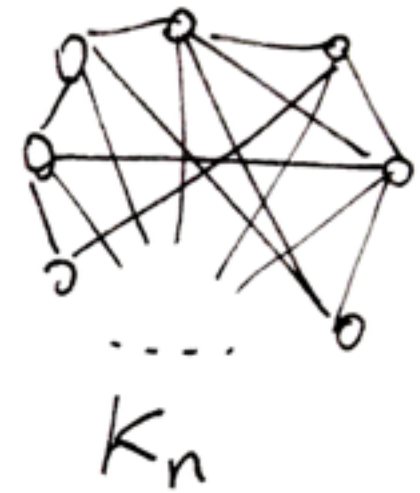
tree decomposition of G

width of decomposition = maximum size of graphs – 1

tree-width of G = minimum width of decomposition of G

Tree-width, examples

(a tree)



tree-width 1

tree-width 2

tree-width $n-1$

Tree-width, examples

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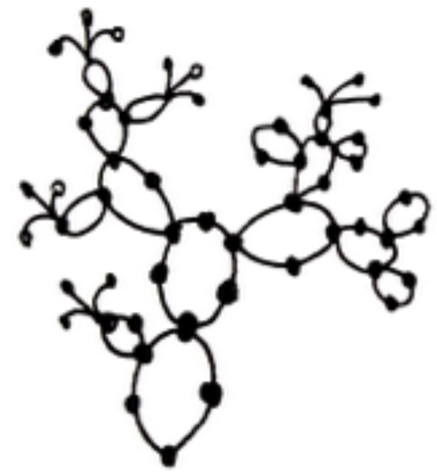

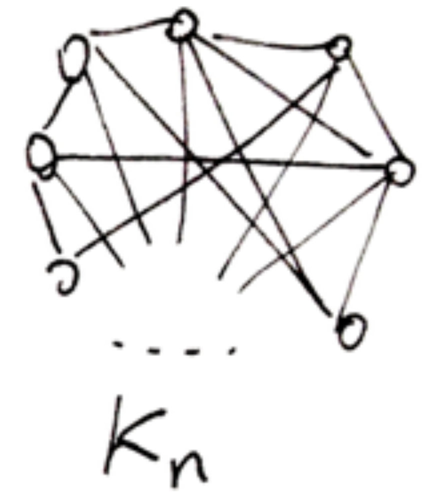

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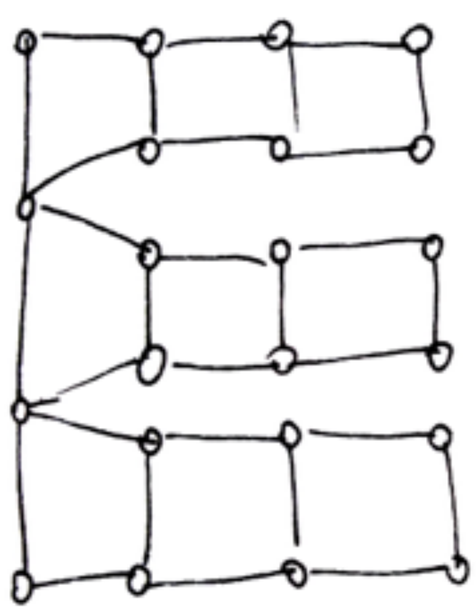

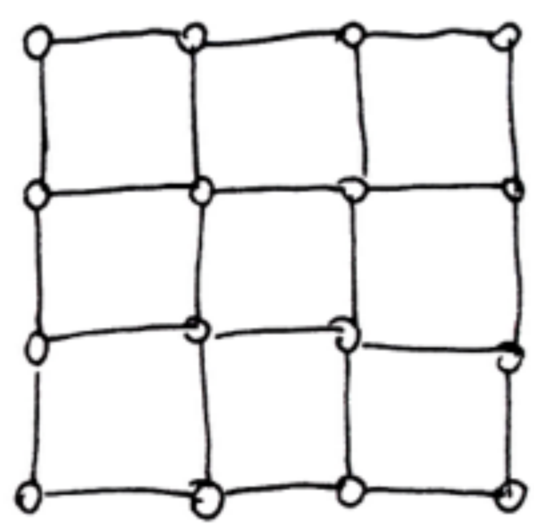

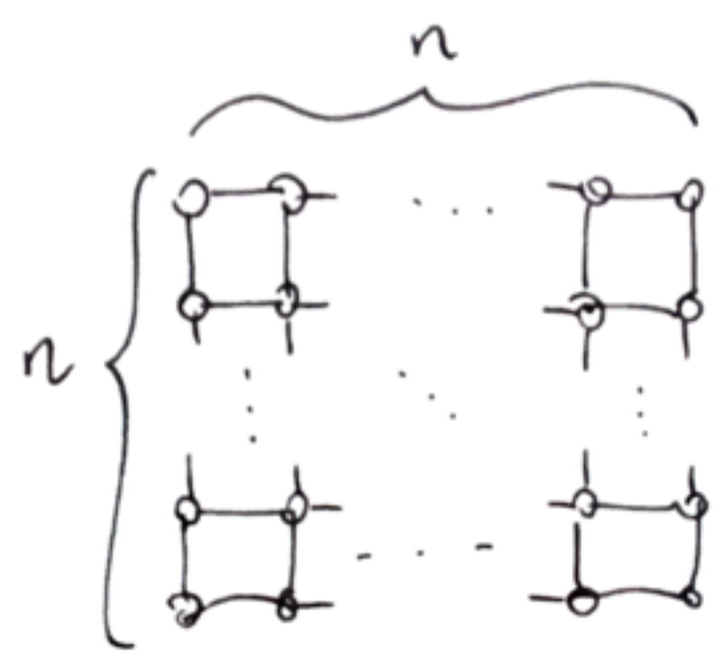

tree-width 2

tree-width 4

tree-width n

Tree-width of structures, queries

tree-width of CQ = tree-width of its canonical structure

tree-width of structure = tree-width of Gaifman graph

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tree-width($\exists\, x_1,x_2,x_3\; R(x_1,x_2) \wedge S(x_1,x_3) \wedge S(x_2,x_3)$)

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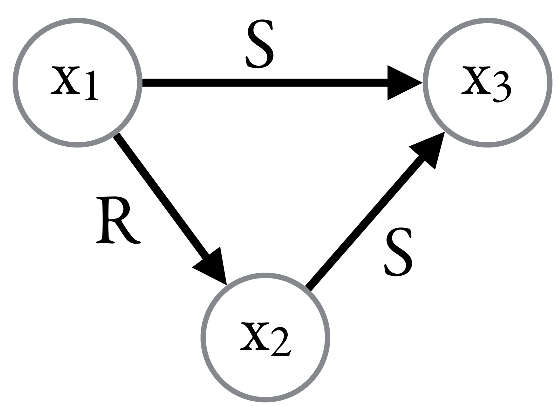
tree-width()

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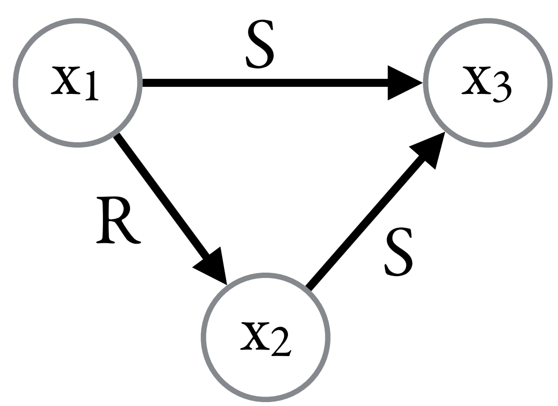

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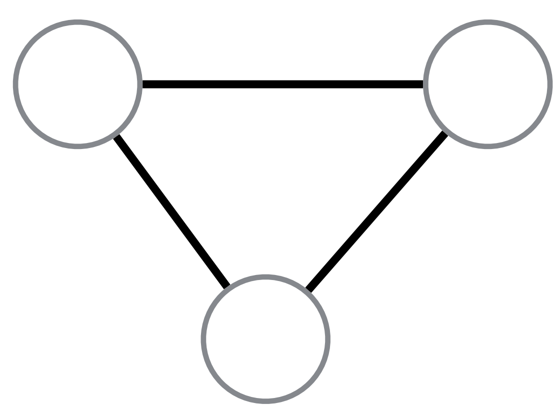


) = 2

Beyond acyclic CQ's

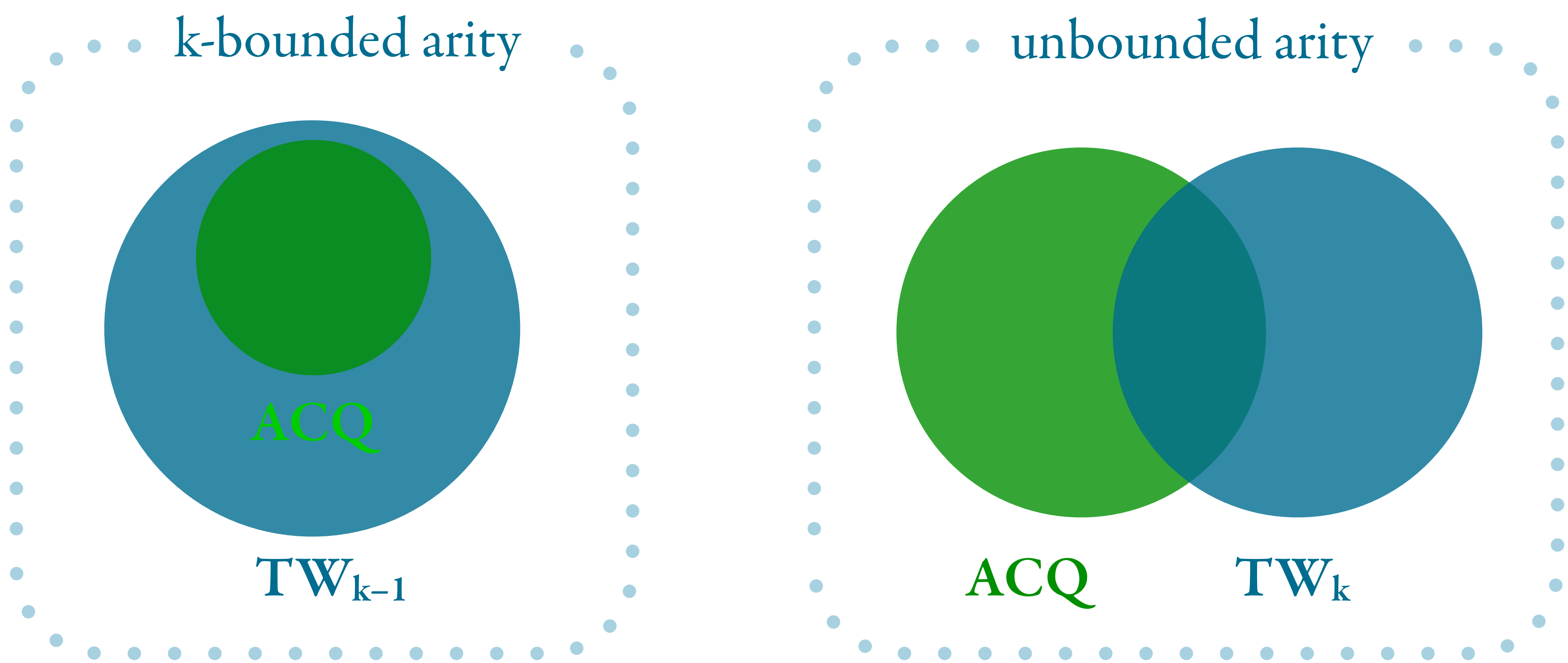
For a fixed k,

- computing whether $\phi \in CQ$ has tw $\leq k$
- calculating a tree decomposition

can be done in **linear time**.

[Bodlaender]

Tree-width vs. Acyclicity



Beyond acyclic CQ's

CQ's with bounded treewidth can be evaluated in PTIME

[Chekuri, Rajaraman, Gottlob, Leone, Scarcello]

Beyond acyclic CQ's

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CQ's can be evaluated in PTIME iff they have bounded tree width!

[Grohe, Schwentick, Segoufin]

Querying with semi-joins

The semi-join

$$R \ltimes_{\{i_1=j_1,\dots,i_n=j_n\}} S = \{ (x_1,\dots,x_n) \in R \mid \text{there is } (y_1,\dots,y_m) \in S \text{ where } x_{i_k} = y_{j_k} \text{ for all } k\}$$

The **semi-join algebra (SA)**: variant of RA with operations:

$\ltimes$, $\cup$, π, σ, $\setminus$, *dupcol*

Output at most linear in the database. Further,

The **evaluation problem** for SA is in $O(|\phi|.|D|)$

Logical characterisation: "stored-tuples guarded fragment of FO"

Acyclic CQs:

- every intermediate relation is **linear** in $|D|$
- we apply $|\phi|$ semi-joins

$\rightsquigarrow$ What if we allow intermediate relations to be **polynomial** in $|D|$?

Bounded variable FO

Def.

$\mathbf{FO}^{k}$ = **The fragment of FO restricted to k variable names**

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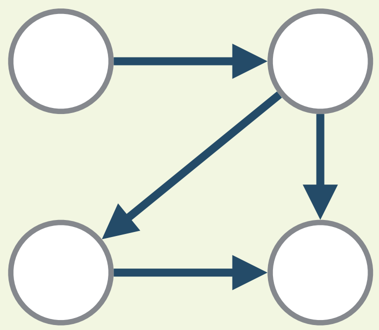
$$= \forall y. \Big(E(x,y) \implies \exists z\, \exists w \big(E(y,z) \wedge E(z,w) \big) \Big) \;\in\; \mathrm{FO}^4$$

G



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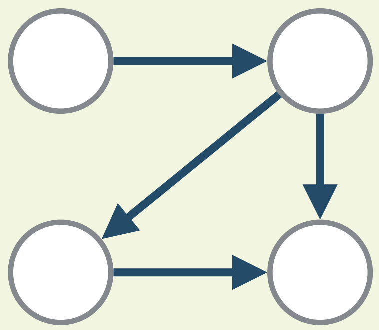
Question: in FO^2 ?

G



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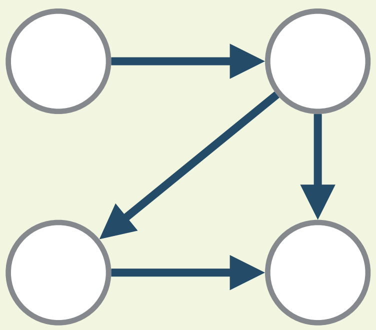

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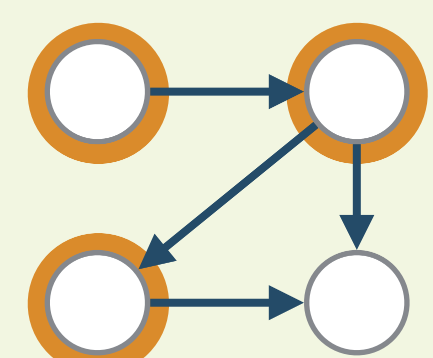

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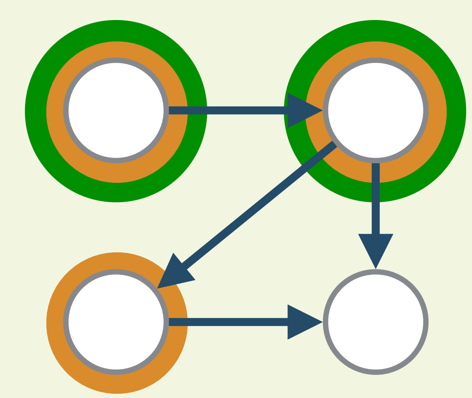

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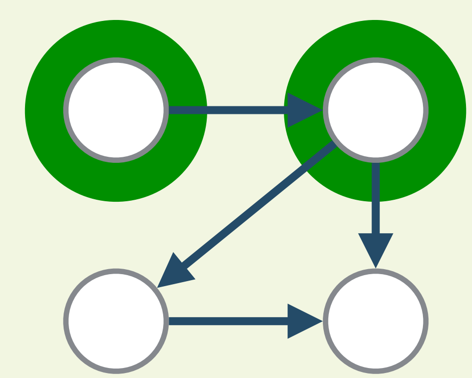

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PTIME

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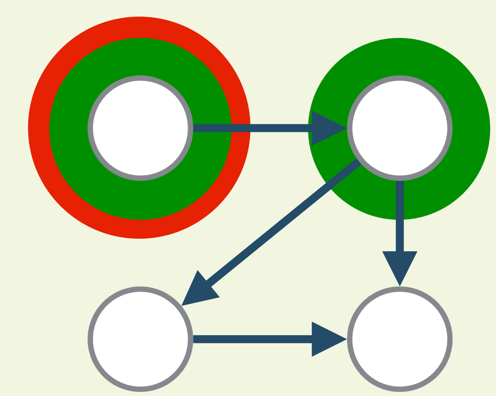

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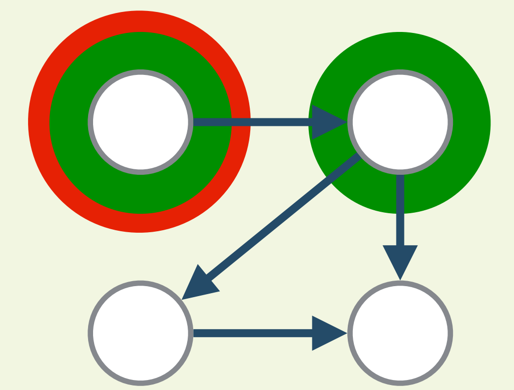

The evaluation problem for FO^k is in PTIME (combined c.)

Bounded variable FO

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Algorithm for a FO^k formula ψ of **quantifier rank** **r**:

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Maximum number of nested quantifiers "$\exists x(\ldots \forall y(\ldots \exists x(\ldots \exists z\underbrace{(\ldots)}_{\text{qr } 0})))$"

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qr 1

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$\ddots$

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2. Evaluate **qr=1** subformulas β based on $\mathbf{R_{0,\alpha}}$ and output in $\mathbf{R_{1,\beta}}$
3. Evaluate **qr=2** subformulas γ based on $\mathbf{R_{1,\beta}}$ and output in $\mathbf{R_{1,\gamma}}$
4. $\ldots$

$\vdots$

r. $\ldots$

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   $\rightsquigarrow |V|^k \cdot (|\beta| \cdot (|G| + \underbrace{|R_1|}_{\leq |V|^k}))^p$

3. Evaluate **qr=2** subformulas γ based on $\mathbf{R_{1,\beta}}$ and output in $\mathbf{R_{1,\gamma}}$

4. …

⋮

r. …

Bounded variable FO

The evaluation problem for FO^k is in PTIME (combined c.)

Maximum number of nested quantifiers "$\exists x(\dots \forall y(\dots \exists x(\dots \exists z(\dots))))$"

qr 0

qr 1

qr 2

$\ddots$

Algorithm for a FO^k formula ψ of **quantifier rank r**:

1. Evaluate **qr=0** subformulas α and output result in relations $\mathbf{R}_{0,\alpha}$

   $\rightsquigarrow |V|^k \cdot (|\alpha| \cdot |G|)^p$

2. Evaluate **qr=1** subformulas β based on $\mathbf{R}_{0,\alpha}$ and output in $\mathbf{R}_{1,\beta}$

   $\rightsquigarrow |V|^k \cdot (|\beta| \cdot (|G| + \underbrace{|R_1|}_{\leq |V|^k}))^p$

3. Evaluate **qr=2** subformulas γ based on $\mathbf{R}_{1,\beta}$ and output in $\mathbf{R}_{1,\gamma}$

   $\rightsquigarrow |V|^k \cdot (|\gamma| \cdot (|G| + \underbrace{|R_2|}_{\leq |V|^k}))^p$

4. $\dots$

$\vdots$

r. $\dots$

Bounded variable FO

Desirable:

- Given k and a FO query ϕ, is ϕ in FO^k ? ⇝ 💀 Undecidable (even w.o. $\neg$)

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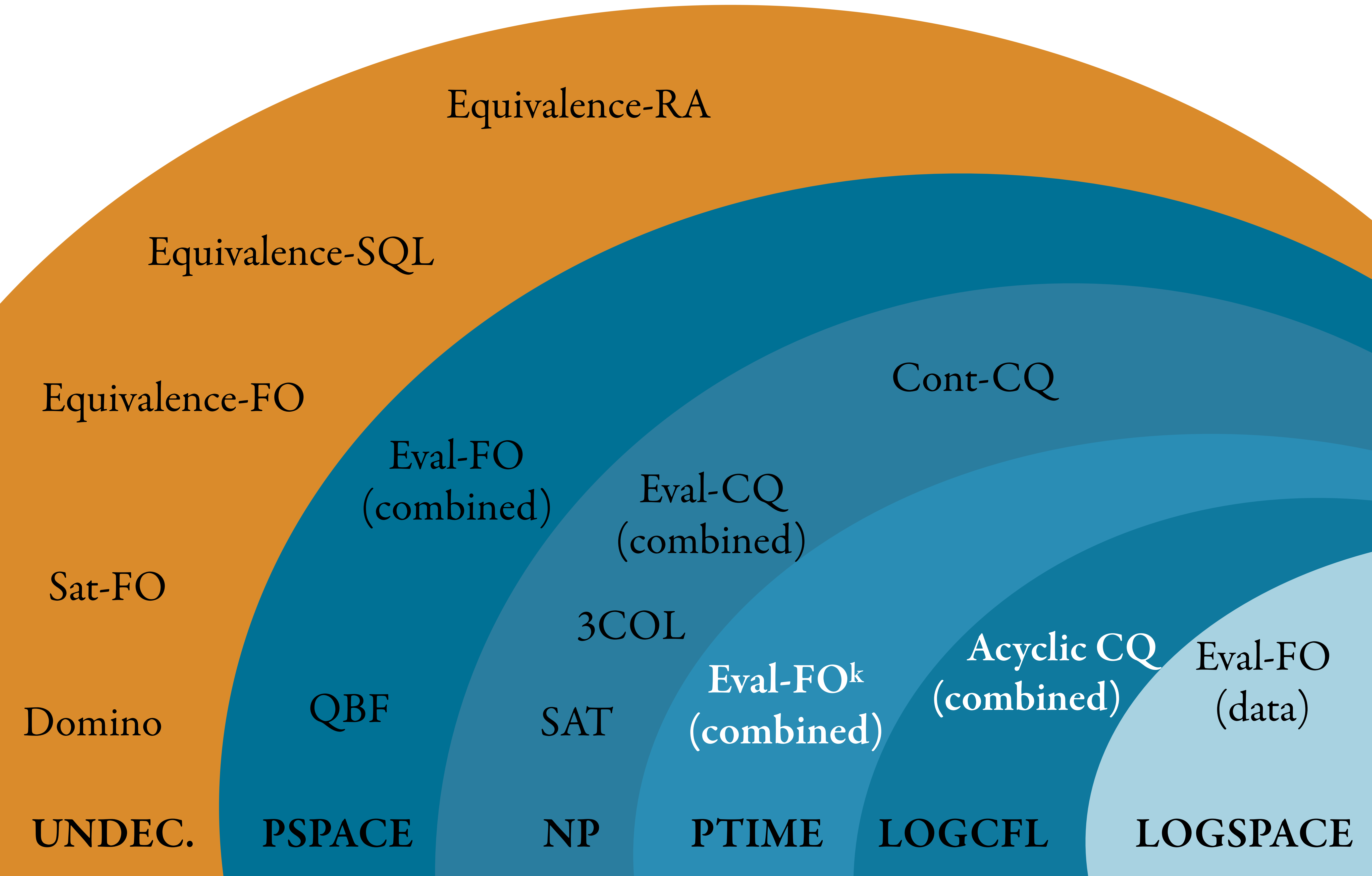
Bounded variable FO

Desirable:

- Given k and a FO query ϕ, is ϕ in FO^k ? $\rightsquigarrow$ 💀 Undecidable (even w.o. $\neg$)
- Given k and a CQ query ϕ, is ϕ in FO^k ? $\rightsquigarrow$ NP-complete
- Satisfiability for FO^k $\rightsquigarrow$ Undecidable if $k \geq 3$ (Domino)

  $\rightsquigarrow$ NEXPTIME-complete if $k=2$

Recap



Equivalence-RA
Equivalence-SQL
Equivalence-FO
Sat-FO
Domino
UNDEC.
Eval-FO (combined)
QBF
PSPACE
Cont-CQ
Eval-CQ (combined)
3COL
SAT
NP
Eval-FO^k (combined)
PTIME
Acyclic CQ (combined)
LOGCFL
Eval-FO (data)
LOGSPACE

Definability in FO

Goal: check which properties / queries are *expressible in FO*

Definability in FO

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Example. $Q(G) = \{\, (u, v) \mid G \text{ contains a path from } u \text{ to } v \,\}$

Is Q expressible as a first-order formula?

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has quantifier rank 3.

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Quantifier rank ≠ quantity of quantifiers

Eg, in $\phi_0(x, y) = E(x, y)$, and

$$\phi_k(x,y) = \exists z \, (\phi_{k-1}(x, z) \wedge \phi_{k-1}(z, y))$$

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Sub-goal: Given a property P and a number n,
tell whether P is expressible by a sentence of quantifier rank at most n.

Definability in FO

Definition. Two structures S_1 and S_2 are ***n*-equivalent** [Tarski '30]

iff

they satisfy the same FO sentences of quantifier rank ≤ n

(i.e. $S_1 \vDash \phi$ iff $S_2 \vDash \phi$ for all $\phi \in$FO with $\mathrm{qr}(\phi) \leq n$)

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Consider a property (i.e. a set of structures) **P**.

Suppose that there are $S_1 \in \mathbf{P}$, $S_2 \notin \mathbf{P}$ *s.t.*

S_1 and S_2 are n*-equivalent*.

Then **P** is *not expressible* by any sentence of quantifier rank n.

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Note: if the above happens $\forall\, n$, then **P** is not expressible by *any* FO sentence.

Example. $\mathbf{P} = \{$ structures of even size $\}$ seems to be not FO-definable.
One could then aim at proving that

for all n there are $S_1 \in P$ and $S_2 \notin P$ s.t. S_1, S_2 n-equivalent...

Expressive power via games

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Characterisation of the expressive power of FO in terms of **Games**

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Characterisation of the expressive power of FO in terms of **Games**

Idea: For every two structures (S,S') there is a game where

a player of the game has a **winning strategy**

iff

S,S' are **indistinguishable**

Ehrenfeucht-Fraïssé games

A game between two players

Board: (S_1, S_2)

One player plays in one structure, the other player answers in the other structure.

If Duplicator can ensure not losing after n rounds: S_1, S_2 are n-equivalent

Ehrenfeucht-Fraïssé games

Definition. **Partial isomorphism** between S_1 and S_2 = injective partial map

$$\mathsf{f} : \text{ nodes of } S_1 \longrightarrow \text{ nodes of } S_2$$

so that $E(x,y)$ **iff** $E(\,\mathsf{f}(x), \mathsf{f}(y)\,)$

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At each round i :

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or

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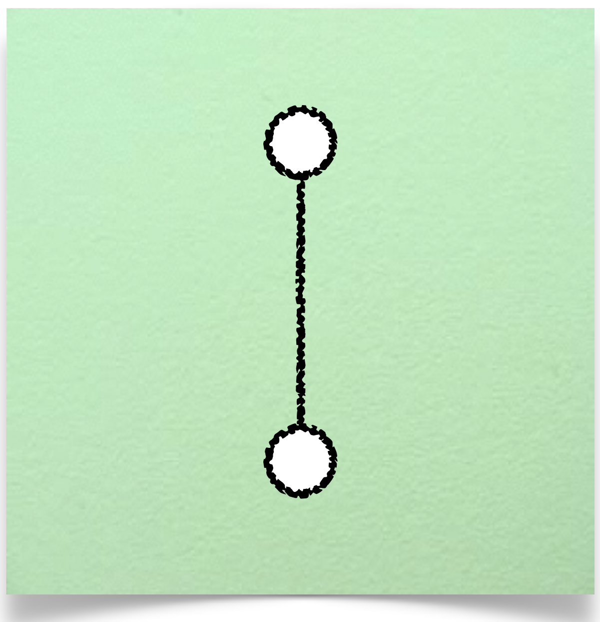
2. Spoiler chooses a node y_i from S_2

   and **Duplicator** answers with a node x_i from S_1,

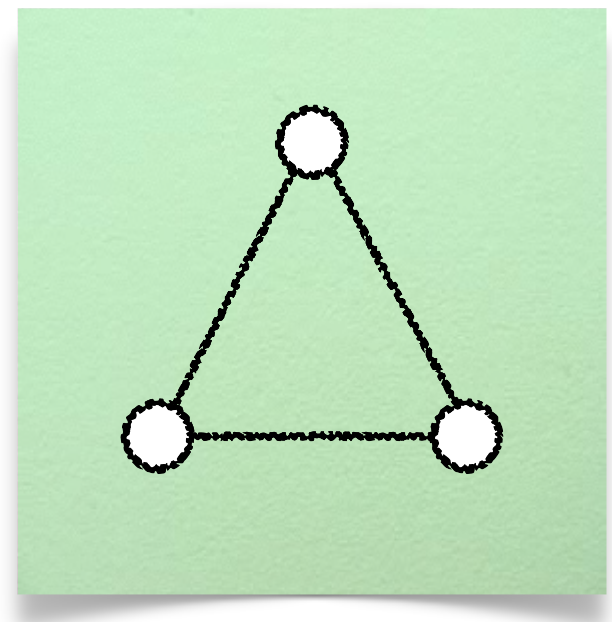
or Spoiler wins if $\{\, x_i \mapsto y_i \mid 1 \leq i \leq n \}$ is **not a partial isomorphism** between S_1 and S_2.

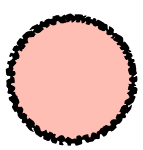
Ehrenfeucht-Fraïssé games

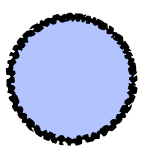
S_1



S_2

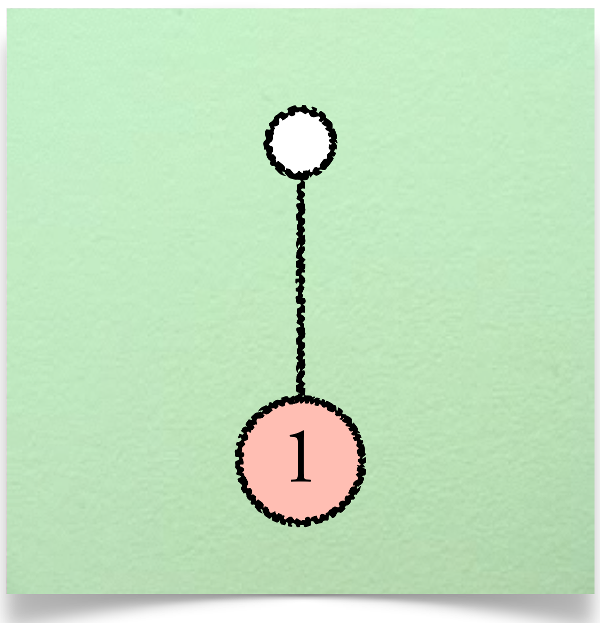


 = Spoiler

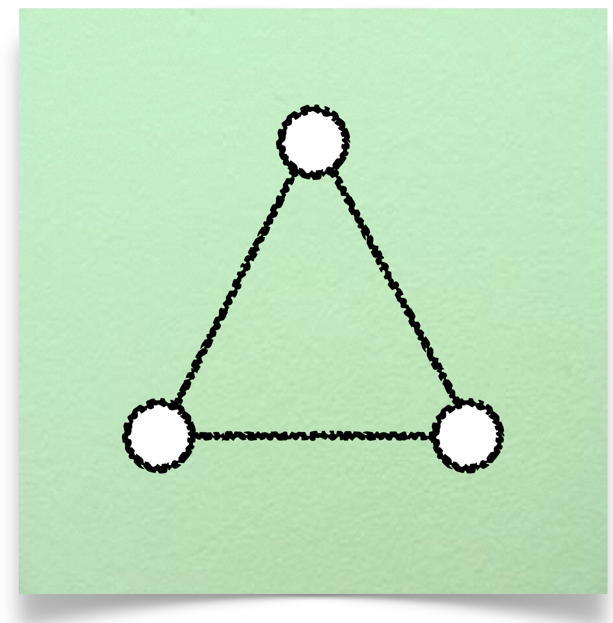
 = Duplicator

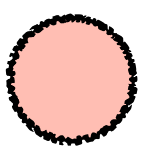
Ehrenfeucht-Fraïssé games

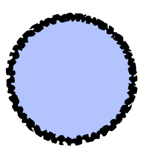
S_1



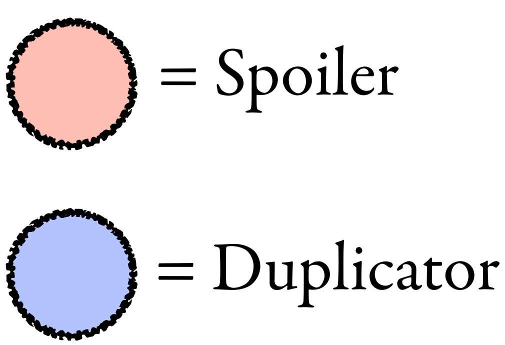
S_2



 = Spoiler

 = Duplicator

Ehrenfeucht-Fraïssé games



Ehrenfeucht-Fraïssé games

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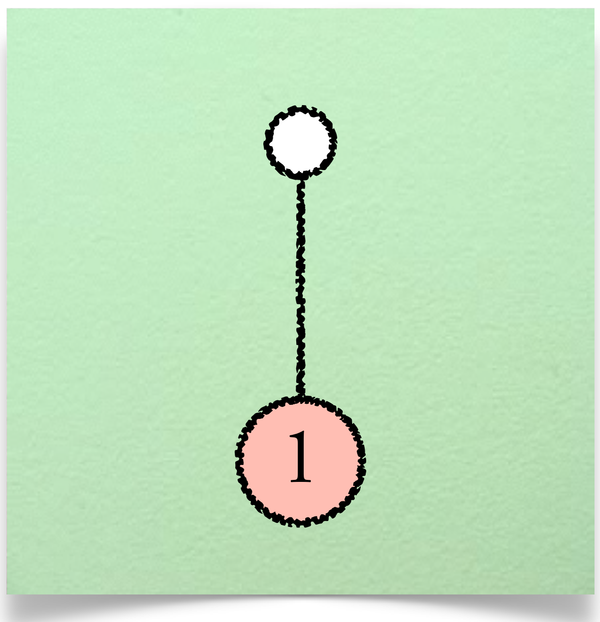
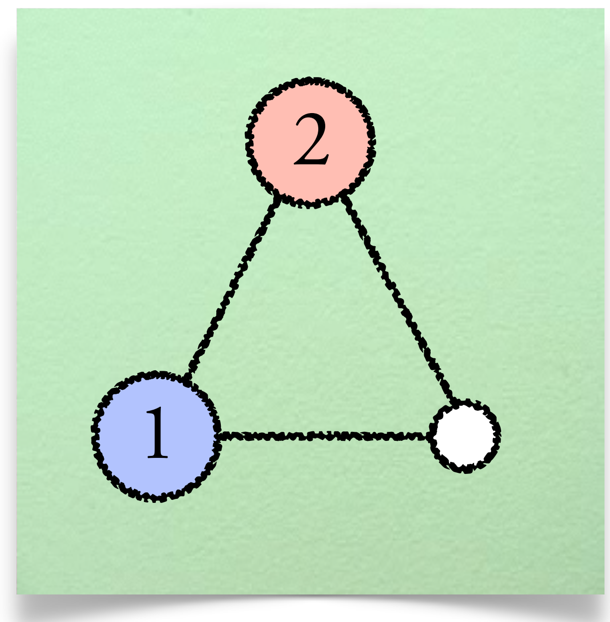
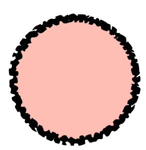
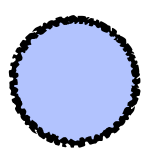


S_2



 = Spoiler

 = Duplicator

Ehrenfeucht-Fraïssé games

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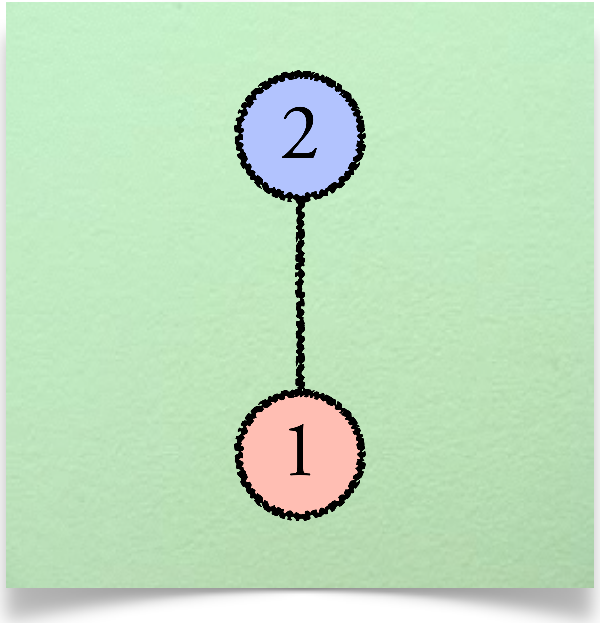
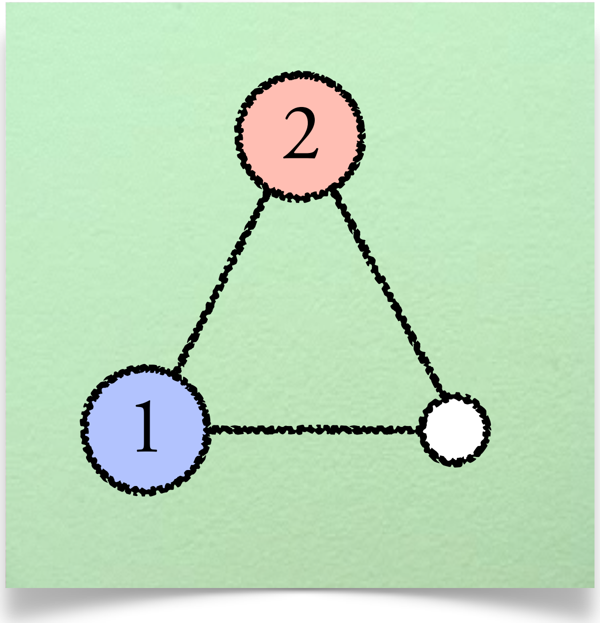
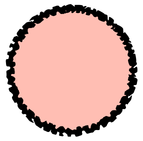
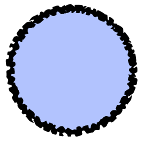


S_2



 = Spoiler

 = Duplicator

Ehrenfeucht-Fraïssé games

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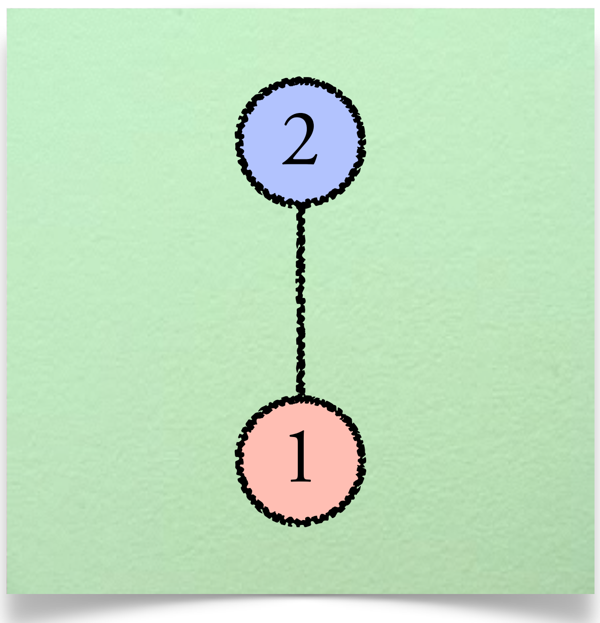
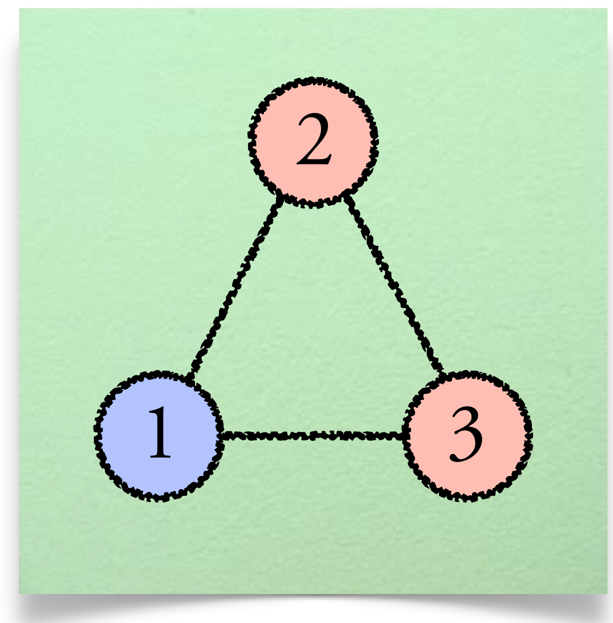
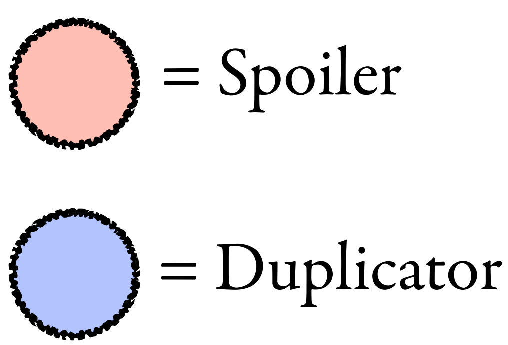


S_2





Ehrenfeucht-Fraïssé games

Question: Can Spoiler win in 3 rounds ?

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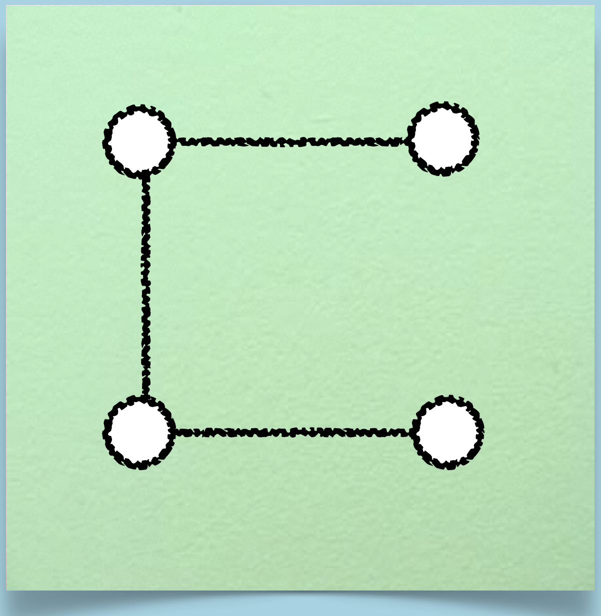

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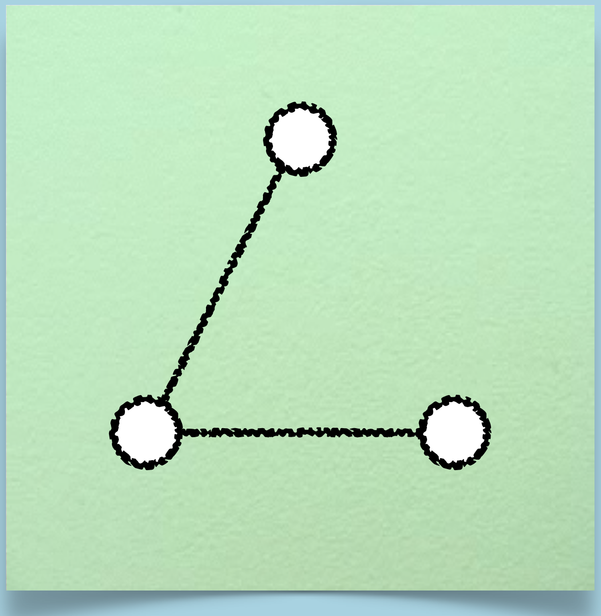
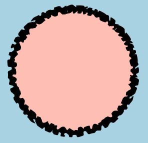
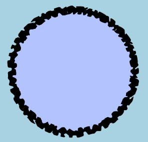


 = Spoiler

 = Duplicator

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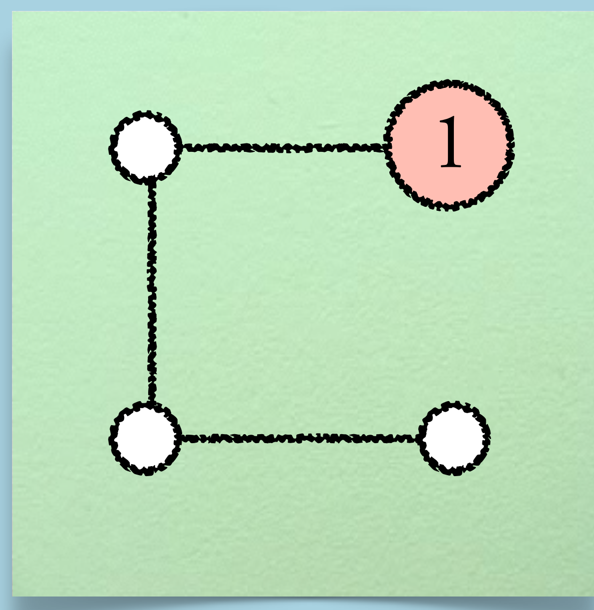
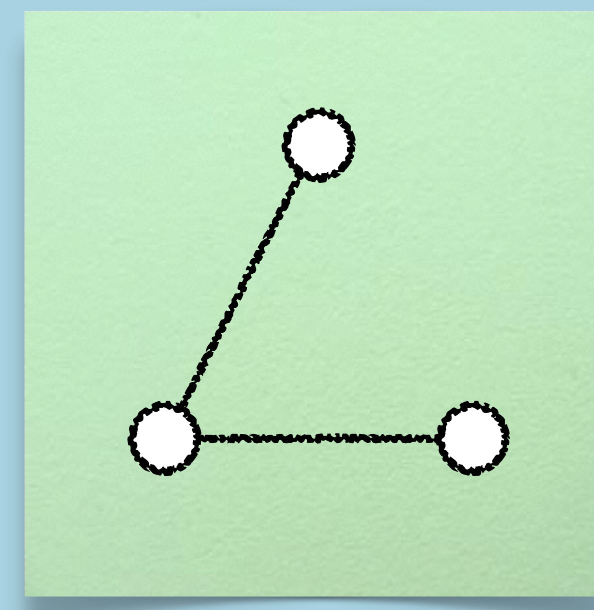
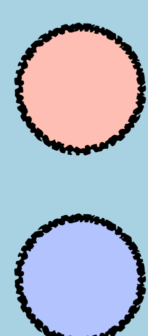


S_2





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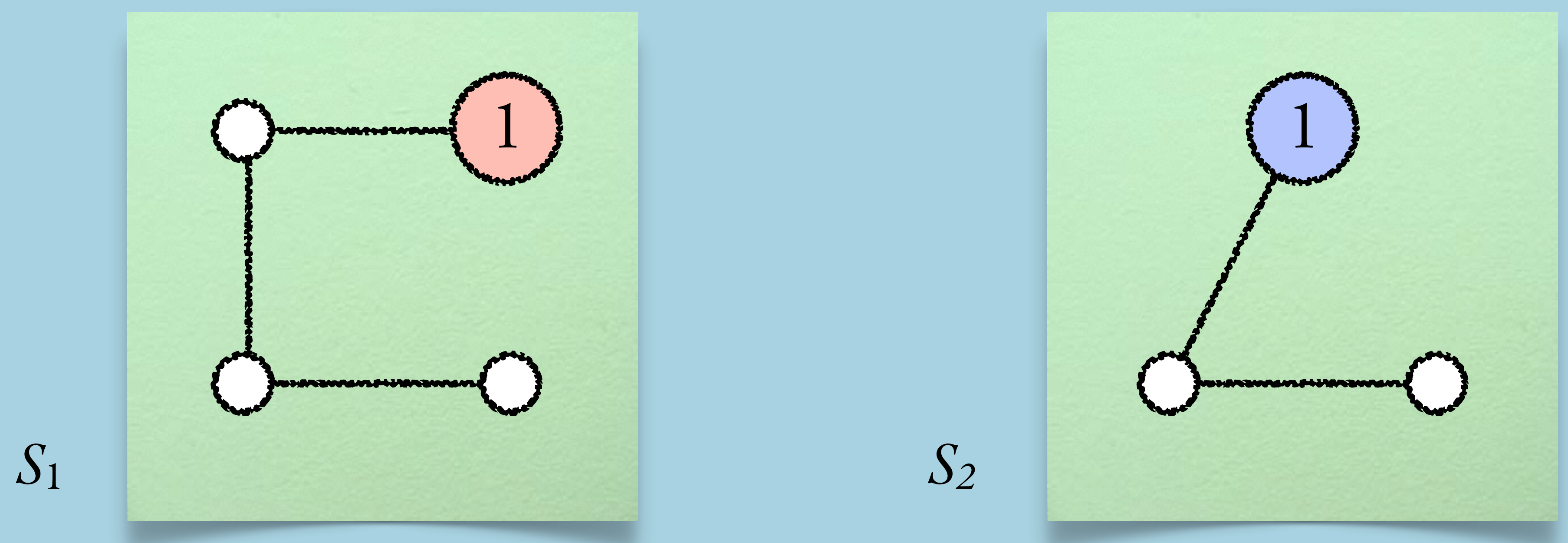

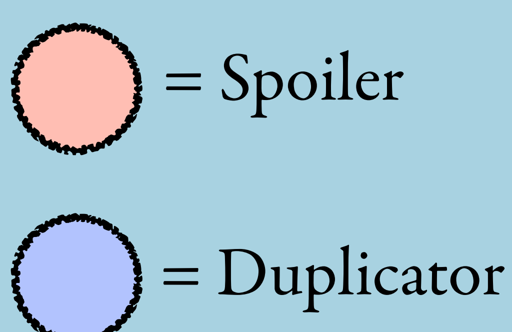

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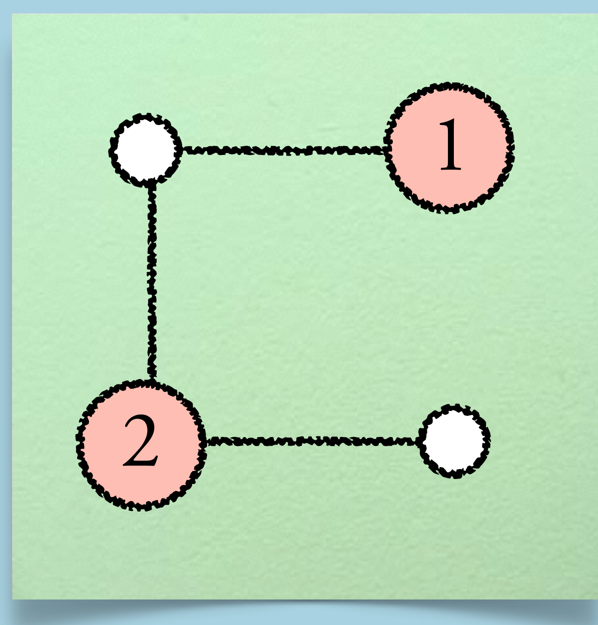
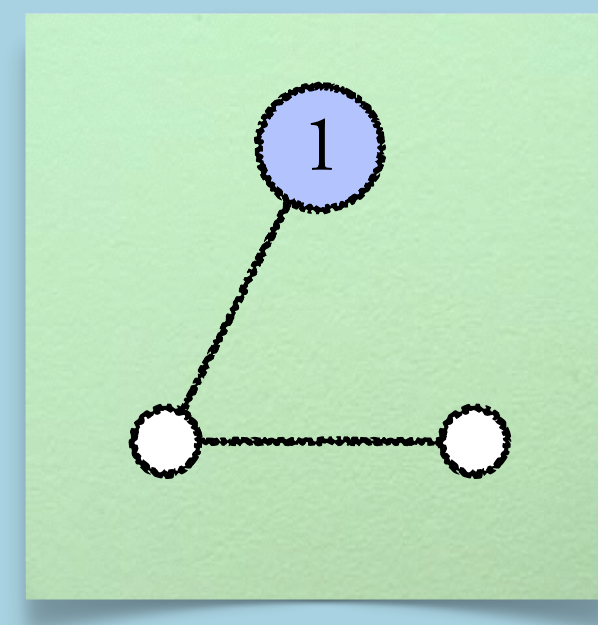
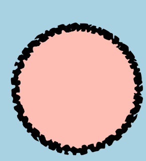
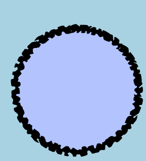


S_2



 = Spoiler

 = Duplicator

Ehrenfeucht-Fraïssé games

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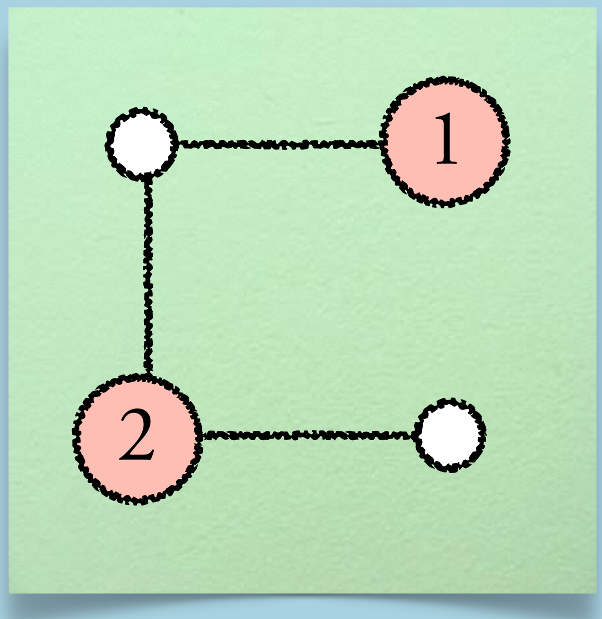
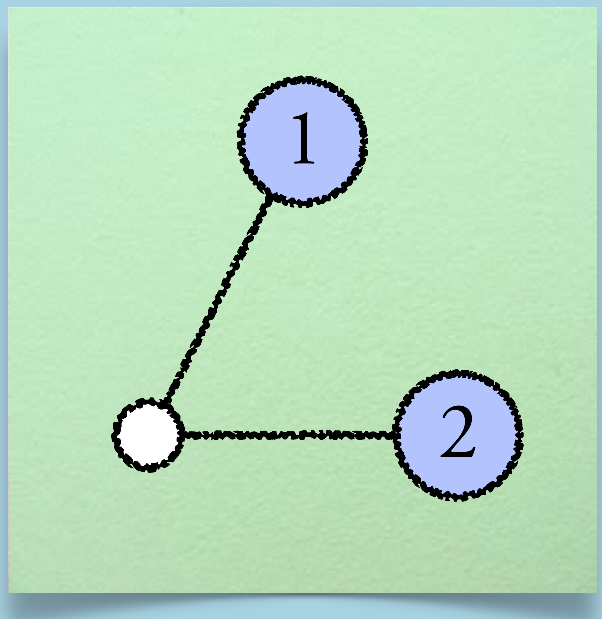
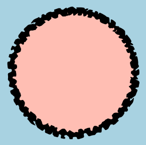
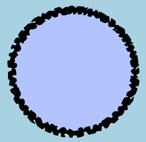


S_2





= Spoiler

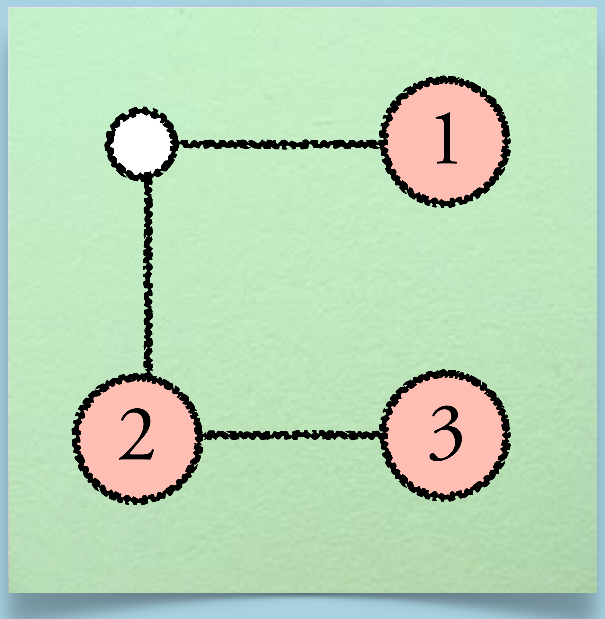


= Duplicator

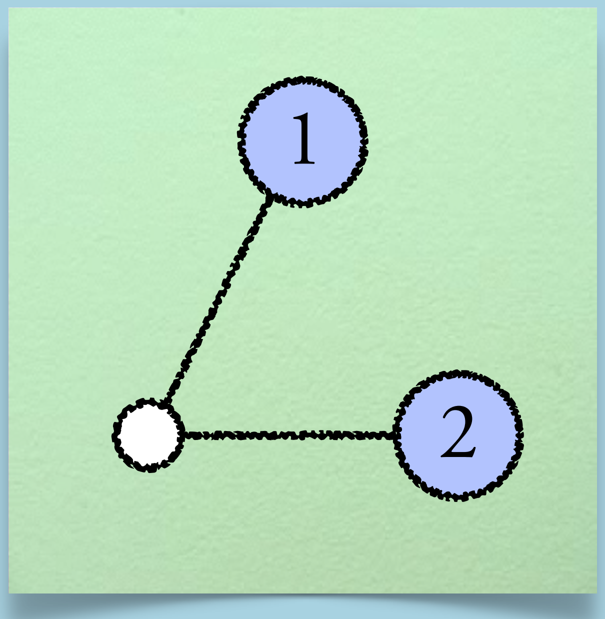
Ehrenfeucht-Fraïssé games

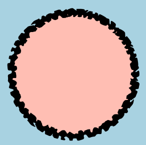
Question: Can Spoiler win in 3 rounds ?

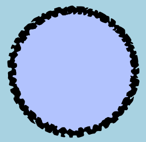
S_1



S_2



 = Spoiler

 = Duplicator