Introduction to Non(-)monotonic Logic

Christian Straßer and Mathieu Beirlaen

Research Group For Non-Monotonic Logic and Formal Argumentation
http://homepages.ruhr-uni-bochum.de/defeasible-reasoning
Institute for Philosophy II
Ruhr University Bochum

christian.strasser@ruhr-uni-bochum.de
mathieubeirlaen@gmail.com

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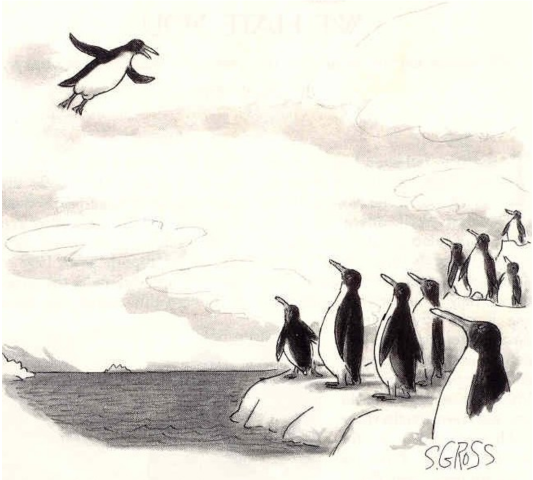
0. Intro



"We just haven't been flapping them hard enough."

Defeasible reasoning and non-monotonic logic

Defeasible reasoning: "jumping to conclusions" which may be withdrawn in the light of new info

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A logic **L** is *monotonic* if, for any formula α and set of formulas Γ:

If $\Gamma \vdash_{\mathbf{L}} \alpha$, then $\Gamma \cup \Delta \vdash_{\mathbf{L}} \alpha$; or

$Cn_{\mathbf{L}}(\Gamma) \subseteq Cn_{\mathbf{L}}(\Gamma \cup \Delta)$

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  Day 5: Formal argumentation

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! Disclaimer: last minute changes

Background (1)

- L is the set of formulas made up of a list of elementary letters and the connectives $\wedge, \vee, \neg$ in such a way that:
  - Elementary letters $p, q, r, \ldots$ are members of L,
  - If α and β are members of L, then so are $\alpha \wedge \beta$ and $\alpha \vee \beta$,
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- An assignment is a function v_a on the set of all elementary letters into the two-element set $\{1, 0\}$.
  Each assignment can be extended uniquely to a valuation which is a function v on the set L into $\{1, 0\}$ that agrees with the assignment on elementary letters and behaves in accord with the standard truth-tables for compound formulas made up using $\wedge, \vee, \neg$:

$v(\alpha) = 1$	iff	$v_a(\alpha) = 1$ (where α is an elementary letter)
$v(\neg\alpha) = 1$	iff	$v(\alpha) = 0$
$v(\alpha \wedge \beta) = 1$	iff	$v(\alpha) = 1$ and $v(\beta) = 1$
$v(\alpha \vee \beta) = 1$	iff	$v(\alpha) = 1$ or $v(\beta) = 1$

Background (2)

$$\alpha \to \beta =_{df} \neg\alpha \vee \beta$$

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Classical consequence is a relation between sets of formulas and individual formulas:

β is a classical consequence of Γ ($\Gamma \models \beta$) iff there is no valuation v such that $v(\Gamma) = 1$ while $v(\beta) = 0$.

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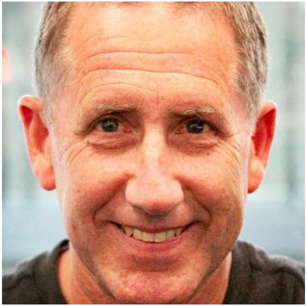
Classical consequence as an operation on sets of formulas: $\mathrm{Cn}(\Gamma) = \{\beta : \Gamma \models \beta\}$.

1. Preferential models á la Shoham



Y. Shoham. A semantical approach to nonmonotonic logics. In M. Ginsberg (ed.), *Readings in Nonmonotonic Reasoning* (Morgan Kaufmann Publishers, 1987), pp. 227-250.

Selecting interpretations: motivation

Bringing nonmonotonic logic closer to standard model theory

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A general account of non-monotonic inference in terms of consequence relations (to do what Tarski did for classical logic)

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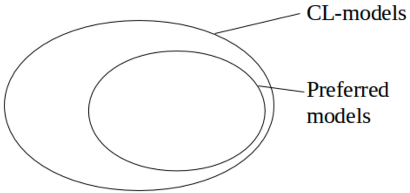

Shoham's idea:

- Interpretations/valuations/models: complete assignments of values to all formulas in L
- Less interpretations $\Rightarrow$ more consequences
- Selecting 'preferrable' models
- Different criteria of preference give rise to different consequence relations
- $\alpha \mid\!\sim \beta$: "β is true in all preferred α-models"

Preferential consequence (Shoham, 1988)

A preferential model is a pair $(V, <)$ where V is a set of interpretations on the language L, and $<$ is an irreflexive, transitive relation over V.

- Irreflexivity: for any $v \in V$, $v \not< v$,
- Transitivity: for any $v_1, v_2, v_3 \in V$, if $v_1 < v_2$ and $v_2 < v_3$, then $v_1 < v_3$.

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Compact notation: $\Gamma \mid\!\sim \beta$ iff $min_<|\Gamma|_V \subseteq |\beta|_V$.

Properties of preferential consequence (1)

Suppose (for a language with elementary letters p, q, r) that $V = \{v_1, v_2\}$, with $v_1 < v_2$

v_2: p, q, ¬r

v_1: p, ¬q, r

$$p, q \mid\!\sim p? \tag{1}$$
$$p \mid\!\sim r? \tag{2}$$
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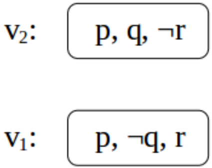

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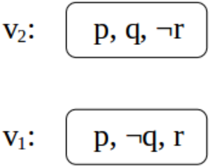

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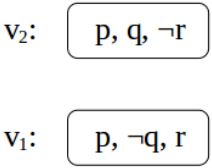

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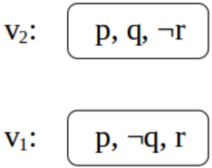

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Properties of preferential consequence (1)

Suppose (for a language with elementary letters p, q, r) that $V = \{v_1, v_2\}$, with $v_1 < v_2$

v_2: $p, q, \neg r$

v_1: $p, \neg q, r$

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- Monotony fails
- Transitivity holds for $<$, but fails for $\mathrel{|\!\sim}$
- What about contraposition for $\mathrel{|\!\sim}$?

Properties of preferential consequence (2)

- ☺ Disjunction in the premisses:

$$\text{If } \Gamma \cup \{\alpha\} \mathrel{|\!\sim} \gamma \text{ and } \Gamma \cup \{\beta\} \mathrel{|\!\sim} \gamma \text{ then } \Gamma \cup \{\alpha \vee \beta\} \mathrel{|\!\sim} \gamma. \qquad \text{(OR)}$$

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Proof. Suppose (first) that $\Gamma \cup \{\alpha\} \mid\!\sim \gamma$ and $\Gamma \cup \{\beta\} \mid\!\sim \gamma$, and (second) that $\Gamma \cup \{\alpha \vee \beta\} \not\mid\!\sim \gamma$. We show that the second supposition leads to a contradiction.

By the second supposition, there is a minimal $\alpha \vee \beta$-valuation v such that $v(\Gamma) = v(\alpha \vee \beta) = 1$ and $v(\gamma) = 0$. Since $v(\alpha \vee \beta) = 1$, we know that $v(\alpha) = 1$ or $v(\beta) = 1$.

- If $v(\alpha) = 1$, then v is a minimal $\Gamma \cup \{\alpha\}$-valuation. (Suppose there is a $v' < v$ such that $v'(\Gamma) = v'(\alpha) = 1$. Then $v'(\alpha \vee \beta) = 1$, contradicting the $\alpha \vee \beta$-minimality of v.) But then, since $v(\gamma) = 0$, $\Gamma \cup \{\alpha\} \not\mid\!\sim \gamma$, contradicting the first supposition.
- If $v(\beta) = 1$, then v is a minimal $\Gamma \cup \{\beta\}$-valuation. (Suppose there is a $v' < v$ such that $v'(\Gamma) = v'(\beta) = 1$. Then $v'(\alpha \vee \beta) = 1$, contradicting the $\alpha \vee \beta$-minimality of v.) But then, since $v(\gamma) = 0$, $\Gamma \cup \{\beta\} \not\mid\!\sim \gamma$, contradicting the first supposition.

Properties of preferential consequence (3)

☺ Cumulative transitivity (Cut):

$$\text{If } \Gamma \mathrel{|\!\sim} \alpha \text{ for all } \alpha \in \Delta \text{ and } \Gamma \cup \Delta \mathrel{|\!\sim} \beta, \text{ then } \Gamma \mathrel{|\!\sim} \beta. \qquad \text{(CT)}$$

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Proof. Suppose $\Gamma \mathrel{|\!\sim} \alpha$ for all $\alpha \in \Delta$ and $\Gamma \not\mathrel{|\!\sim} \beta$. We show that $\Gamma \cup \Delta \not\mathrel{|\!\sim} \beta$. Since $\Gamma \not\mathrel{|\!\sim} \beta$ there is a minimal Γ-valuation v with $v(\beta) = 0$. Since $\Gamma \mathrel{|\!\sim} \alpha$ for all $\alpha \in \Delta$, $v(\alpha) = 1$ for all $\alpha \in \Delta$. Hence v is an $\Gamma \cup \Delta$-valuation. Suppose now that there is an $\Gamma \cup \Delta$-valuation $v' < v$. Then $v'(\Gamma) = 1$ contradicting the fact that v is a minimal Γ-valuation. Hence v is a minimal $\Gamma \cup \Delta$-valuation with $v(\beta) = 0$, so $\Gamma \cup \Delta \not\mathrel{|\!\sim} \beta$. □

Properties of preferential consequence (4)

? Cautious monotony:

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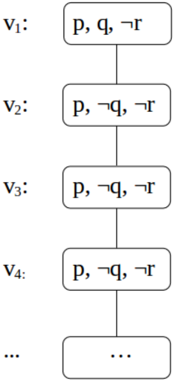

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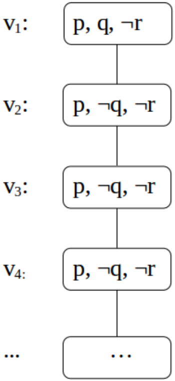

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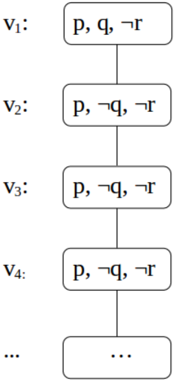

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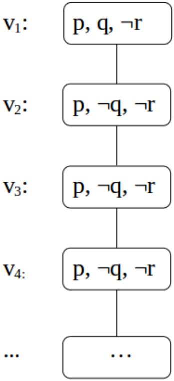

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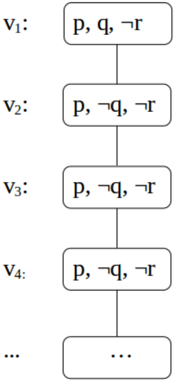

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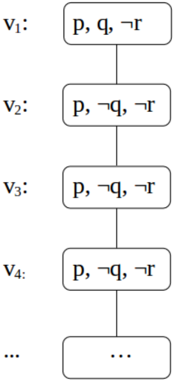

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- (CM) fails in general
- (CM) holds for models that satisfy the *smoothness* or *stopperedness* condition:

Smoothness condition:
If $v \in |\Gamma|_V$ then (either $v \in min_<|\Gamma|_V$ or there is a $u < v$ with $u \in min_<|\Gamma|_V$).

Summary: preferential consequence

Given a preferential model $(V, <)$, β is a preferential consequence of Γ ($\Gamma \mathrel{|\!\sim} \beta$) iff $v(\beta) = 1$ for every interpretation $v \in V$ that is minimal among those in V that satisfy Γ.

- Monotony fails
- Transitivity fails
- Contraposition fails
- Disjunction in the premisses holds
- Cumulative transitivity holds
- Cautious monotony fails (but holds for smooth models)

2. Preferential models á la Kraus, Lehmann, and Magidor

  

S. Kraus, D. Lehmann, and M. Magidor.
Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial Intelligence* 44(1-2), 1990, pp. 167–207.

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- Additional expressive power added for providing representation results
- $\alpha \mid\!\sim \beta$ means 'if α, then normally β' or 'β is a plausible consequence of α'
- Concrete elaboration of Shoham's idea that "different criteria of preference give rise to different consequence relations" (systems **C**, **P** and **R**)

Shoham-models with worlds and states

Equivalently to Shoham's definitions:

- Let $\prec$ be a preference relation on worlds $w_1, w_2, \ldots \in W$ so that $w_1 \prec w_2$ means that w_1 is more normal than w_2. Worlds are complete and respect classical truth-conditions.

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- $\prec$ now relates states instead of worlds.
- The same set of worlds can appear twice in the ordering as the label of different states. This adds expressive power to Shoham's construction.

Cumulative consequence (semantically)

A model is a triple $\langle S, l, \prec \rangle$ where

- S is a set of states
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$\alpha \mid\!\sim \beta$ (relative to a cumulative model M) iff
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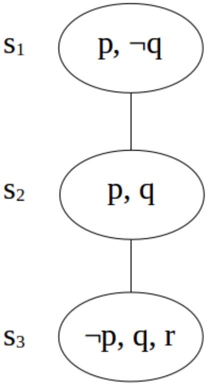
Cumulative consequence (illustration)

We are given a language with elementary letters p, q, r and the following model:



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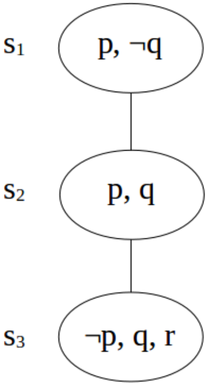

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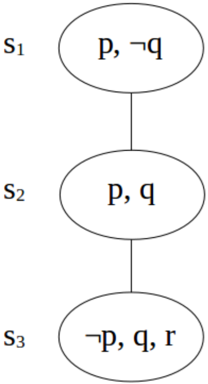

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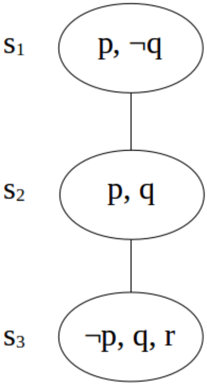

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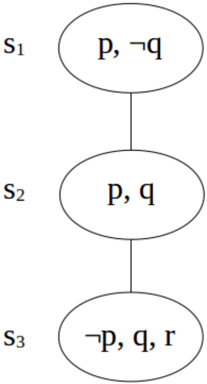

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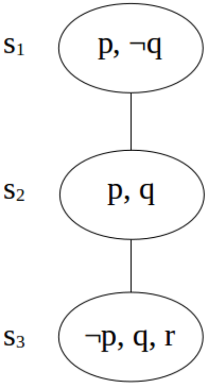

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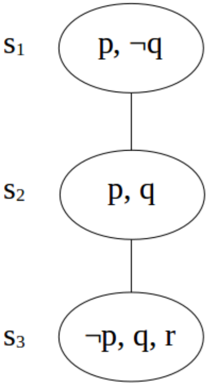

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Transitivity and contraposition fail.

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A consequence relation $\mid\!\sim$ is cumulative iff it contains all instances of the reflexivity axiom (R) and is closed under the inference rules of left logical equivalence (LLE), right weakening (RW), cut (CT), and cautious monotony (CM):

$$\alpha \mid\!\sim \alpha \qquad \text{(R)}$$

$$\frac{\models \alpha \leftrightarrow \beta, \quad \alpha \mid\!\sim \gamma}{\beta \mid\!\sim \gamma} \qquad \text{(LLE)} \qquad\qquad \frac{\alpha \wedge \beta \mid\!\sim \gamma, \quad \alpha \mid\!\sim \beta}{\alpha \mid\!\sim \gamma} \qquad \text{(CT)}$$

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Rules (not) derivable in system **C**

DERIVABLE

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$$\frac{\alpha \mid\!\sim \beta \to \gamma, \quad \alpha \mid\!\sim \beta}{\alpha \mid\!\sim \gamma} \quad \text{(MPC)}$$

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$$\frac{\alpha \mid\!\sim \beta \to \gamma}{\alpha \wedge \beta \mid\!\sim \gamma} \quad \text{(EHD)}$$

$$\frac{\alpha \mid\!\sim \beta, \quad \beta \mid\!\sim \gamma}{\alpha \mid\!\sim \gamma} \quad \text{(TRA)}$$

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Note: (EHD) + (RW) makes $\mid\!\sim$ monotonic

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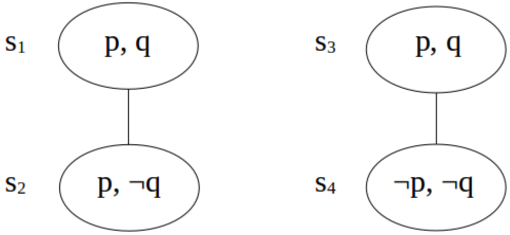
  Let L range over two variables p and q. The following model has no equivalent model in which no label appears twice:



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Preferential consequence: illustration

- p for 'penguin', b for 'bird', f for 'flies'
- Our knowledge base Γ contains

$$p \mid\!\sim b \tag{4}$$

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- Note that:

$$p \not\mid\sim f \tag{7}$$
$$p \wedge b \mid\sim \neg f \tag{8}$$
$$f \mid\sim \neg p \tag{9}$$
$$b \mid\sim \neg p \tag{10}$$
$$b \vee p \mid\sim f \wedge \neg p \tag{11}$$

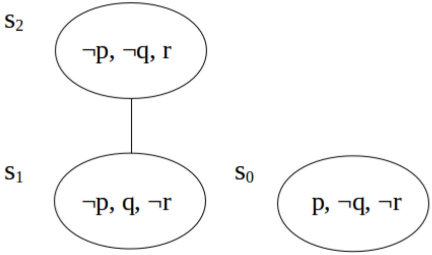
3. Discussion

"We think a good reasoning system should validate all rules of **P**"
"Nevertheless, many preferential reasoners lack properties that seem desirable"
(KLM, 1990)

Rational Monotony fails in **P**

$$\frac{\alpha \mid\!\sim \gamma, \quad \alpha \not\mid\!\sim \neg\beta}{\alpha \wedge \beta \mid\!\sim \gamma} \qquad \text{(Rational Monotony)}$$

Counter-example (Lehmann & Magidor, 1992):

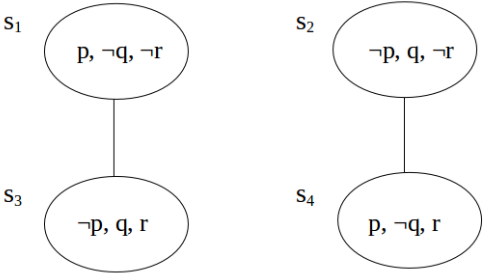


$$p \vee q \vee r \mid\!\sim \neg r$$
$$p \vee q \vee r \not\mid\!\sim \neg\neg q$$
$$(p \vee q \vee r) \wedge \neg q \not\mid\!\sim \neg r$$

Disjunctive Rationality fails in **P**

$$\frac{\alpha \not\mathrel{|}\!\sim \gamma, \quad \beta \not\mathrel{|}\!\sim \gamma}{\alpha \vee \beta \not\mathrel{|}\!\sim \gamma} \qquad \text{(Disjunctive Rationality)}$$

Counter-example (Makinson, 1988):



$$p \not\mathrel{|}\!\sim r$$
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Negation Rationality fails in **P**

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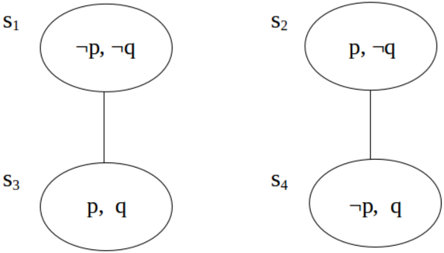

$p \not\mid\!\sim q$

$\neg p \not\mid\!\sim q$

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System **R**: ranked models

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A consequence relation $\mid\!\sim$ is ranked iff it satisfies all rules of **P** plus the rule of Rational Monotony.

System **R**: ranked models

A ranked model $\langle S, l, \prec \rangle$ is a preferential model for which the strict partial order $\prec$ is modular, i.e.

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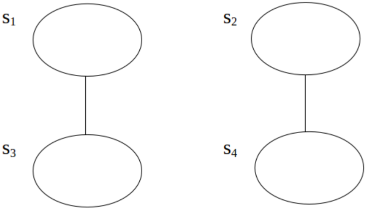
Disjunctive Rationality and Negation Rationality are derivable in **R**.



$s_3 \not\prec s_2$

$s_2 \not\prec s_1$

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