Logics of agency

Chapter 2: Propositional Dynamic Logic and Theory of Action

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- Most of this course will consider logics of agency “proper”.
- They generally abstract away from action names.
- Different from Davidson’s treatment.
- Different from Dynamic Logic (PDL and variants).
- This chapter thus briefly covers PDL and its applications to theories of action.

Propositional Dynamic Logic (PDL) [Hoare 1969], [Pratt 1976], [Harel et al. 2001]:

Language:

- names for atomic events
- complex events are built recursively by means of imperative programming constructs
  - ";" (sequential composition),
  - "∪" (nondeterministic composition),
  - "*" (iteration),
  - "?" (test), ...

Example: the event of "felling a tree by performing the atomic 'chop' action until the tree is down":

$$\pi_{\mathit{fellTree}} = (\neg \mathit{treeDown}?; \mathbf{chop})^*; \mathit{treeDown}?$$

The event “felling a tree”:

$$\pi_{\mathit{fellTree}} = (\neg \mathit{treeDown}?; \mathbf{chop})^*; \mathit{treeDown}?$$

Equivalent:

```
1: if (not treeDown)
2:     chop
3:     goto 1
4: else
5:     done
```

Equivalent:

```
while (not treeDown)
  chop
```

- if φ then α else $\beta =_{\textit{def}} ((\varphi?;\alpha) \cup (\neg\varphi?;\beta))$
- while φ do $\alpha =_{\textit{def}} ((\varphi?;\alpha)^*;\neg\varphi?)$
- repeat α until $\varphi =_{\textit{def}} (\alpha;((\varphi?;\alpha)^*;\neg\varphi?))$
- abort $=_{\textit{def}} \bot?$
- skip $=_{\textit{def}} \top?$

Application to the theory of action:

- [Cohen & Levesque 1990]
- [van Linder et al. 96-99] (KARO)

Mental attitudes, time, ..., intention.

Outline

1. Propositional Dynamic Logic

2. Theory of intentional action

Language of PDL

Terms:

- atomic propositions Prop $= \{p, q, r, \ldots, p_1, p_2, \ldots\}$
- atomic events Evt $= \{\alpha, \beta, \ldots, \alpha_1, \alpha_2, \ldots\}$

In general, an event has the form:

$$\pi ::= \alpha \mid \varphi? \mid \pi;\pi \mid \pi \cup \pi \mid \pi^*$$

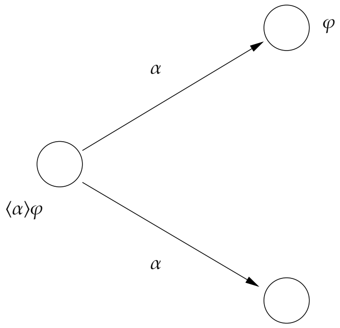
where φ is a proposition (see next), and $\alpha \in$ Evt.

The language (the set of all propositions / well-formed sentences) is given by the grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle\pi\rangle\varphi$$

where $p \in$ Prop and π is an event.

$\langle\alpha\rangle\varphi$ means that "there is a possible course of event α that yields the proposition φ".



A PDL model is a tuple $\langle W, R, V \rangle$:

- W: a set of possible worlds
- R: where $R_\alpha \subseteq W \times W$ for all $\alpha \in$ Evt
- V: is a valuation function $V(p) \subseteq W$ for all $p \in$ Prop

$(w, v) \in R_\alpha$: "there is a course of α from w that ends in v"

- We can extend R naturally to general events π: $R_\pi \subseteq W \times W$.
- We can extend V naturally to general propositions φ:
  - $w \models p$ iff $w \in V(p)$
  - $w \models \langle\pi\rangle\varphi$ iff there is $v \in W$ such that $(w, v) \in R_\pi$ and $v \models \varphi$

Necessity:

$$[\pi]\varphi = \neg\langle\pi\rangle\neg\varphi$$

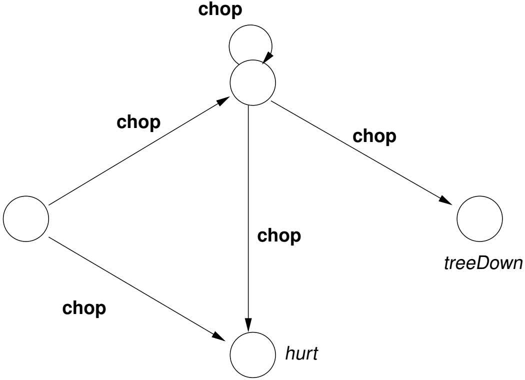
Possible execution:

$$\langle\pi\rangle\top$$

Impossible execution:

$$[\pi]\bot$$

$\pi_{fellTree} = (\neg treeDown?; \textbf{chop})^*; treeDown?$



In the left-most world:

1. $\langle \textbf{chop}; \textbf{chop} \rangle treeDown$
2. $\neg \langle \textbf{chop} \rangle treeDown$
3. $\langle \textbf{chop}; \textbf{chop}; \textbf{chop} \rangle hurt$
4. $\langle \pi_{fellTree} \rangle \top$
5. $\langle \pi_{fellTree} \rangle treeDown \wedge [\pi_{fellTree}] treeDown$
6. $\langle \pi_{fellTree} \rangle \neg treeDown$ never holds

- *hurt*? is a state event
- **chop*** is an activity event
- π_{fellTree} is an accomplishment event that necessarily culminates in world satisfying *treeDown*

Outline

[Bratman 1986]

- Intentions pose problems for the agent; the agent needs to determine a way to achieve them.
- Intentions provide a “screen of admissibility” for adopting other intentions.
- Agents “track” the success of their attempts to achieve intentions.

[Cohen & Levesque 1987, 1990] have adapted PDL models to capture actual action and intention.

Intention is choice with commitment: intention is a composite specifying what an agent choose and is commited to.

(We use the slightly simplified presentation of [Herzig & Longin 2004] [Meyer, Broersen, Herzig 2012].)

PDL SPECIALIZATION

Linear PDL:

- a world has at most one immediate successor
- more than one atomic action can yield the transition
- formally: if $(u, v_1) \in R_\alpha$ and $(u, v_2) \in R_\beta$ then $v_1 = v_2$

In this context, we use:

- $Happ_\pi\varphi =_{def} \langle\pi\rangle\varphi$
- $(IfHapp_\pi\varphi =_{def} \neg Happ_\pi\neg\varphi = [\pi]\varphi)$

We let Agt to be a set of agents.

An atomic event is now an object referring to an agent and a action he does. E.g., *i* doing α:

$$i : \alpha$$

Quantification over actions:

$$\exists \alpha Happ_{i:\alpha} \varphi$$

Future tense modalities (textbook: [Baier, Katoen 2008]):

$$\varphi U \psi \quad / \quad F\varphi \quad / \quad G\varphi$$

Beliefs:

$$Bel_i \varphi$$

Choices/Realistic Preferences (originally *Goal*):

$$Choose_i \varphi$$

i can make φ happen:

$$\exists\alpha Happ_{i:\alpha}\varphi$$

φ is true until ψ true / φ is true eventually / φ is always true:

$$\varphi U\psi \quad / \quad F\varphi \quad / \quad G\varphi$$

i believes φ:

$$Bel_i\varphi$$

i chooses φ to be true (realistic preference):

$$Choose_i\varphi$$

For every atomic event (action) $i : \alpha$ and $j : \beta$:

$$(Happ_{i:\alpha}\varphi \land Happ_{j:\beta}\top) \rightarrow Happ_{i:\beta}\varphi$$

- $Bel_i\varphi \rightarrow \neg Bel_i\neg\varphi$ (consistency, axiom D)
- $Bel_i\varphi \rightarrow Bel_iBel_i\varphi$ (positive introspection, axiom 4)
- $\neg Bel_i\varphi \rightarrow Bel_i\neg Bel_i\varphi$ (negative introspection, axiom 5)

- $Bel_i\varphi \rightarrow Choose_i\varphi$ (realism)

(We did not give the semantic constraints of Bel_i and $Choose_i$; they are standard in modal logics: R_{Bel_i} is serial, transitive, Euclidean. $R_{Choose_i} \subseteq R_{Bel_i}$.)

Achievement goal:

$$AGoal_i\varphi = Choose_i F\varphi \wedge Bel_i\neg\varphi$$

Persistent goal:

$$PGoal_i\varphi = AGoal_i\varphi \wedge (AGoal_i\varphi) U (Bel_i\varphi \vee Bel_i G\neg\varphi \vee \psi)$$

(ψ is a “superior” reason for abandoning the goal. “Mom told me I shouldn’t do it.” Not present in [Herzig & Longin 2004].)

Intention:

$$Intend_i\varphi = PGoal_i\varphi \wedge Bel_i F\exists\alpha Happ_{i:\alpha}\varphi$$

Valid principles

if $\models \varphi \leftrightarrow \psi$ then $\models \textit{Intend}_i\varphi \leftrightarrow \textit{Intend}_i\psi$

$\models \textit{Intend}_i\varphi \rightarrow \textit{Bel}_i\neg\varphi$

...

Nice:

$$\not\models \textit{Intend}_i(\varphi \wedge \psi) \rightarrow \textit{Intend}_i\varphi \wedge \textit{Intend}_i\psi$$

$$\not\models \textit{Intend}_i\varphi \wedge \textit{Intend}_i\psi \rightarrow \textit{Intend}_i(\varphi \wedge \psi)$$

Nice:

$$\not\models (\textit{Intend}_i\varphi \wedge \textit{Bel}_i(\varphi \rightarrow \psi)) \rightarrow \textit{Intend}_i\psi$$

(e.g., if I intend to go to the dentist and believe that going to the dentist will cause pain then I do not necessarily intend to have pain)

Not very nice:

$$\not\models \textit{Intend}_i\varphi \rightarrow \textit{Bel}_i\textit{Intend}_i\varphi$$

$$\not\models \neg\textit{Intend}_i\varphi \rightarrow \textit{Bel}_i\neg\textit{Intend}_i\varphi$$

Variant: obtaining introspective intention

Let us program introspective choices into the logic:

- $Choose_i\varphi \rightarrow Bel_i Choose_i\varphi$
- $\neg Choose_i\varphi \rightarrow Bel_i \neg Choose_i\varphi$

These become valid:

- $Intend_i\varphi \rightarrow Bel_i Intend_i\varphi$
- $\neg Intend_i\varphi \rightarrow Bel_i \neg Intend_i\varphi$

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