Logics of agency

Chapter 5: Applications of agency to Power

Nicolas Troquard

ESSLLI 2016 – Bolzano

- Remarks about power and deemed ability
- Power in Coalition Logic and in STIT, also with imperfect information
- Advanced: resource-sensitive agency and ability

Necessary Agency → Contingent Power

Power/ability is very much studied in philosophy. So I'll try my best to give the best pointers.

- Philosophy on power / free-will
- Using BIAT's agency to define evidence-based ability
- Ability in concurrent games: CL / ATL
- Ability in STIT

Outline

What power is not

Ayers [Ayers 68] identified three approaches to power that he considered fallacious.

- Transcendentalists
- Skeptics
- Reductionists

I. Transcendentalists

- Power is an occult entity.
- transcendentalist doctor: the virtue of opium to put people to sleep by the fact that it possesses the virtus dormitivae.
- transcendentalist mechanics: looking into the engine of a car, he would expect to see the horsepower.
- In profane terms, this view obliges us to explain that some acting entity has the ability to do something because “it has what it takes”.
- This view tends to obscure the difference between judgments like ‘this is red’ and ‘this can lift ten tons’. But we don’t observe powers as we observe qualities.

II. SKEPTICS

- No evidence is adequate to give knowledge that power exists.
- There certainly is a strong notion of power for which this is true.
- Not helpful for any practical purpose.
- Many commonsense notions of power have grounds in evidence of ability.
- "Being deemed able" is one of them. A professional golf player can be deemed able to sink an easy putt.

III. Reductionists

- III.(i) Power is nothing but its exercise.
- In reaction to transcendentalism Hume believed that it can be said that an agent has the power to do φ when and only when they are actually φing (like the Megarians before him).

> *The distinction, which we often make betwixt power and the exercise of it, is equally without foundation. (Treatise [Hume 1888, Sec XIV])*

- A golf player is deemed able to sink the putt at some time if and only if he does actually sink the putt at that time.
- Ignores the dispositional nature of power.
- Aristotle remarked in particular that if we assimilate power with its exercise, the concepts of art, skill, learning, forgetting disappear.

- III.(ii) Power is nothing but its vehicle.
- Power is its vehicle (agent).
- W.V.O. Quine: power refers to the “subdivisible structure” that is shared by every vehicle of that power: Attractive ontological approach.
- In a system of limited size, the structure of the vehicles will depend on only a few variables. It is often easy to meaningfully characterise what is the vehicle of some power. E.g, to have the ability to read a file owned by *userx* is equivalent to be being logged on as *userx* or as *root*.
- In larger systems, an immediate challenge is that many entities with very different structures can have the same power: a bucket of water, a cold wind, a quantity of pyrene foam, all possess the same power to extinguish a flame [Morriss 1987, p. 18].

Outline

A VERY WEAK ACCOUNT

[Elgesem 1997] proposes

$$E_a\varphi \to C_a\varphi$$

It is compatible with the Megarian reductionism.

[Troquard 2014] does not go much further for coalitional power:

$$E_X\varphi \wedge E_Y\psi \to C_{X\cup Y}\varphi$$

We can try to use time to make it stronger.

Deemed ability

We will assume a linear flow of time and the abstract modal notions of evidence and falsification.

1. If the current situation provides evidence that x is able to bring about φ then x is deemed able to bring about φ;
2. If the current situation falsifies that x is able to bring about φ then x is not deemed able to bring about φ;
3. If an acting entity x is deemed able to bring about φ, it will be deemed able until we encounter a situation that falsifies this ability, or the falsification is not an eventuality;
4. If an acting entity x is not deemed able to bring about φ, it remains so until a situation is reached that provides evidence for it, or the evidence is not an eventuality;
5. If an acting entity x is deemed able to bring about something, it is so because there is evidence of it now, or there has been evidence of it in past and x has been deemed able ever since.

We will use three linguistic constructs that are at the core of the logic of being deemed able.

- $\text{CAN}_G\varphi$ reads "acting entity G is deemed able to bring about that φ".
- $\text{EVID}_G\varphi$ reads "the situation is evidence that acting entity G is able to bring about that φ".
- $\text{FALS}_G\varphi$ reads "the situation falsifies that acting entity G is able to bring about that φ".

Formally, we obtain use the following language L_{sc} (where $p \in \text{Prop}$ and $G \subseteq \text{Agt}$):

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathsf{CAN}_G\varphi \mid \mathsf{EVID}_G\varphi \mid \mathsf{FALS}_G\varphi$$

[prop] an axiomatisation of classical propositional logic
[sc1] $\vdash_{sc} \mathsf{EVID}_G\varphi \rightarrow \mathsf{CAN}_G\varphi$
[sc2] $\vdash_{sc} \mathsf{FALS}_G\varphi \rightarrow \neg\mathsf{CAN}_G\varphi$
[scr1] if $\vdash_{sc} \varphi \leftrightarrow \psi$ then $\vdash_{sc} \mathsf{CAN}_G\varphi \leftrightarrow \mathsf{CAN}_G\psi$
[scr2] if $\vdash_{sc} \varphi \leftrightarrow \psi$ then $\vdash_{sc} \mathsf{EVID}_G\varphi \leftrightarrow \mathsf{EVID}_G\psi$
[scr3] if $\vdash_{sc} \varphi \leftrightarrow \psi$ then $\vdash_{sc} \mathsf{FALS}_G\varphi \leftrightarrow \mathsf{FALS}_G\psi$

DEFINITION

An sc-model is a tuple $M = \langle W, \mathit{dabl}, \mathit{evid}, \mathit{fals}, V\rangle$, where for every $w \in W$ and $G \subseteq \mathrm{Agt}$, $\mathit{dabl}(w)(G) \subseteq \mathcal{P}(W)$, $\mathit{evid}(w)(G) \subseteq \mathcal{P}(W)$, $\mathit{fals}(w)(G) \subseteq \mathcal{P}(W)$, and $V(w) \subseteq \mathrm{Prop}$. In addition, it satisfies the following constraints:

1. if $X \in \mathit{evid}(w)(G)$ then $X \in \mathit{dabl}(w)(G)$
2. if $X \in \mathit{fals}(w)(G)$ then $X \notin \mathit{dabl}(w)(G)$

We define the interpretation $\models_{sc}$ of the language L_{sc} in an *sc*-model $M = \langle W, dabl, evid, fals, V \rangle$ as follows:

- $M, w \models_{sc} p$ iff $p \in V(w)$
- $M, w \models_{sc} \neg\varphi$ iff not $M, w \models_{sc} \varphi$
- $M, w \models_{sc} \varphi \wedge \psi$ iff $M, w \models_{sc} \varphi$ and $M, w \models_{sc} \psi$
- $M, w \models_{sc} \mathsf{CAN}_G\varphi$ iff $||\varphi||^M \in dabl(w)(G)$
- $M, w \models_{sc} \mathsf{EVID}_G\varphi$ iff $||\varphi||^M \in evid(w)(G)$
- $M, w \models_{sc} \mathsf{FALS}_G\varphi$ iff $||\varphi||^M \in fals(w)(G)$

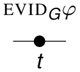
where $||\varphi||^M = \{w \mid M, w \models_{sc} \varphi\}$.

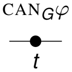
It is routine to prove that the logic sc is a sound and complete wrt. to the class of sc-models.

PROPOSITION

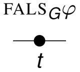
Let $\varphi \in L_{sc}$. Then, $\vdash_{sc} \varphi$ iff $\models_{sc} \varphi$.

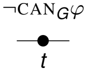
Effectively, the two constraints correspond to imposing the static principle linking an evidence at an instant to a deemed ability at that instant

if 

then 

and the static principle linking a falsification at an instant to an absence of deemed ability at that instant.

if 

then 

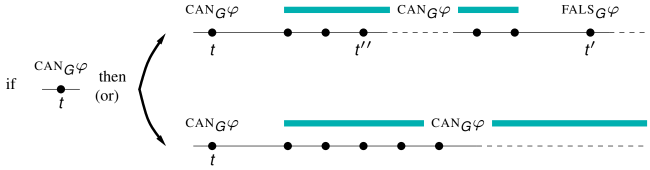
- Temporalization of the static core logic [Finger & Gabbay 1992, Th. 2.3]

$$\varphi \quad ::= \quad \alpha \quad | \quad \neg\varphi \quad | \quad \varphi \wedge \varphi \quad | \quad \varphi \mathsf{U} \varphi \quad | \quad \varphi \mathsf{S} \varphi$$

  where α is a (monolithic) formula of L_{sc}.
- Definition of "weak until": $\varphi \mathsf{W} \psi = (\varphi \mathsf{U} \psi) \vee \mathsf{G}\varphi$
- Some additional axioms...

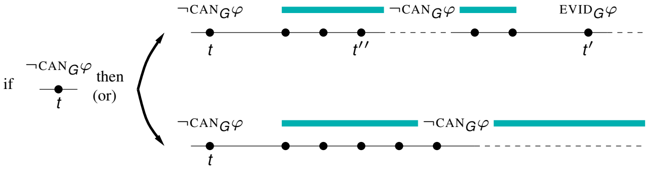
When an acting entity is deemed able of something, one can maintain this perceived ability until some further evidence falsifies it.

$$\vdash_{\mathsf{lbda}} \mathsf{CAN}_G\varphi \rightarrow (\mathsf{CAN}_G\varphi) W (\mathsf{FALS}_G\varphi) \qquad (\textbf{sdc1})$$



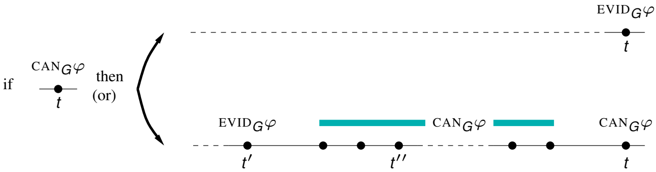
If an acting entity is not deemed able to bring about something, how do we maintain this inability? We adopt the following principle, that is symmetrical to sdc1.

$$\vdash_{\text{lbda}} \neg\mathsf{CAN}_G\varphi \rightarrow (\neg\mathsf{CAN}_G\varphi)W(\mathsf{EVID}_G\varphi) \qquad \textbf{(sdc2)}$$



It remains to address what must be the past chronicle of an existing ability. An entity G is deemed able of φ only if it has been so ever since the occurrence of a situation showing evidence for it.

$$\vdash_{\mathrm{lbda}} \mathrm{CAN}_G\varphi \rightarrow (\mathrm{EVID}_G\varphi) \vee ((\mathrm{CAN}_G\varphi) S (\mathrm{EVID}_G\varphi)) \ (\textbf{sdc3})$$



MELE'S SIMPLE ABILITY

Mele [Mele 2003, p. 448] distinguishes S-ability from I-ability.

- simple ability to *A*: "an agent's *A*-ing at a time is sufficient for his having a simple ability to *A* at that time"
- ability to *A* intentionally: "being able to *A* intentionally entails having a simple ability to *A* and the converse is false."

We do not address *I*-ability. *S*-ability is already captured by [Elgesem 1993, 1997].

In [Elgesem 1993, 1997]

$$E_G\varphi \rightarrow \text{CAN}_G\varphi$$

is an axiom

$$\neg E_G\varphi \wedge \text{CAN}_G\varphi$$

is consistent

Agency-grounded evidence:

$$\vdash E_G\varphi \rightarrow \text{EVID}_G\varphi \qquad \textbf{(b4)}$$

$$\vdash E_{G_1}\varphi \wedge E_{G_2}\psi \rightarrow \text{EVID}_{G_1 \cup G_2}(\varphi \wedge \psi) \qquad \textbf{(b5)}$$

Attempts: [Santos et al. 1997]

$$\text{if } \vdash \varphi \leftrightarrow \psi \text{ then } \vdash Att_G\varphi \leftrightarrow Att_G\psi \qquad \textbf{(br2)}$$

Attempt-grounded falsifications: *x*'s ability to bring about some proposition φ is *x*'s power to bring about φ when *x* tries [Kenny 1975].

$$\vdash Att_G\varphi \wedge \neg E_G\varphi \rightarrow \text{FALS}_G\varphi \qquad \textbf{(b7)}$$

The following deductions can be drawn.

1. If group G is not deemed able to do φ at some time, $\neg\text{CAN}_G\varphi$, axiom sdc2 makes sure that it is so until some evidence occurs.
2. Suppose at some later time some acting entities $G_1, \ldots G_k$ bring about respectively $\varphi_1, \ldots \varphi_k$ such that $\vdash \varphi_1 \wedge \ldots \wedge \varphi_k \leftrightarrow \varphi$. By axiom b5 and rule scr2 one can deduce $\text{EVID}_G\varphi$.
3. By axiom sc1 one can deem G able to bring about φ: $\text{CAN}_G\varphi$.
4. By axiom sdc1, G will be deemed able of doing φ until some falsification occurs.
5. Suppose that at some later time, G attempts to bring about φ but does not actually bring it about, then by axiom b7 one can infer a falsification: $\text{FALS}_G\varphi$.
6. By axiom sc2, we infer that G is not deemed able to bring about φ: $\neg\text{CAN}_G\varphi$, and the life cycle is back to step 1.

Outline

Strategic games and effectivity

Definition

A *strategic game* is a tuple $G = (S, \{\Sigma_i | i \in \mathsf{Agt}\}, o)$ where S is a nonempty set, Σ_i is a nonempty set of choices for every agent $i \in \mathsf{Agt}$, $o : \prod_{i \in \mathsf{Agt}} \Sigma_i \longrightarrow S$ is an outcome function which associates an outcome state in S with every combination of choice of agents (choice profile).

Effectivities of coalitions in strategic games can be defined as the subsets of S that they can ensure.

Definition

Given a strategic game G, the *effectivity function* $E_G : \mathcal{P}(\mathsf{Agt}) \longrightarrow \mathcal{P}(\mathcal{P}(S))$ *of* G is defined as $X \in E_G(C)$ iff there is $\sigma_C \in \prod_{i \in C} \Sigma_i$ such that for every $\sigma_{\overline{C}} \in \prod_{i \in \overline{C}} \Sigma_i$ we have $o(\sigma_C \times \sigma_{\overline{C}}) \in X$.

DEFINITION (TRULY PLAYABLE EFFECTIVITY FUNCTION)

An effectivity function $E : \mathcal{P}(Agt) \longrightarrow \mathcal{P}(\mathcal{P}(S))$ is said to be *truly playable* iff

1. $\forall J \subseteq Agt, \emptyset \notin E(J)$;
2. $\forall J \subseteq Agt, S \in E(J)$;
3. E is Agt-maximal; (if $\overline{X} \notin E(\emptyset)$ then $X \in \overline{\text{Agt}}$)
4. E is outcome-monotonic;
5. E is superadditive;
6. E is $\emptyset$-complete. (for every $X \in E(C)$ there is $Y \in E^{nc}(C)$ such that $Y \subseteq X$)

CHARACTERIZING POWERS OF COALITIONS

There is a strong link between playable effectivity functions and strategic games.

THEOREM (PAULY 2001, GORANKO, JAMROGA, TURRINI 2010)

An effectivity function E is truly playable iff it is the effectivity function of some strategic game.

Semantics of Coalition Logic

Definition

A *coalition model* is a tuple (S, E, V) where:

- S is a nonempty set of states;
- $E : S \longrightarrow (\mathcal{P}(\text{Agt}) \longrightarrow \mathcal{P}(\mathcal{P}(S)))$ is called an *effectivity structure* and for all s, $E(s)$ is a truly playable effectivity function;
- $V : S \longrightarrow \mathcal{P}(\text{Prop})$ is a valuation function.

$$M, s \models \langle\![J]\!\rangle\varphi \text{ iff } \{s \mid M, s \models \varphi\} \in E(s)(J)$$

Axiomatics of Coalition Logic

- Propositional Logic
- $\langle\![J]\!\rangle\top$
- $\neg\langle\![J]\!\rangle\bot$
- $\neg\langle\![\emptyset]\!\rangle\neg\varphi \rightarrow \langle\![Agt]\!\rangle\varphi$
- $\langle\![J]\!\rangle(\varphi \wedge \psi) \rightarrow \langle\![J]\!\rangle\varphi$
- $\langle\![J_1]\!\rangle\varphi \wedge \langle\![J_2]\!\rangle\psi \rightarrow \langle\![J_1 \cup J_2]\!\rangle(\varphi \wedge \psi)$, $J_1 \cap J_2 = \emptyset$
- if $\vdash \varphi \leftrightarrow \psi$ then $\vdash \langle\![J]\!\rangle\varphi \leftrightarrow \langle\![J]\!\rangle\psi$

Outline

$\Diamond[a\ \textit{astit} : \varphi]$ does not capture any kind of power.

$\Diamond[a\ \textit{cstit} : \varphi]$ does.

How to embed Coalition Logic?

A DISCRETE-DETERMINISTIC STIT

HYPOTHESIS (DISCRETENESS)

Given a moment m_1, there exists a successor moment m_2 such that $m_1 < m_2$ and there is no moment m_3 such that $m_1 < m_3 < m_2$.

$m/h \models \mathbf{X}\varphi$ iff φ is true at the moment immediately after m on h

HYPOTHESIS (DETERMINISM)

$\forall m \in Mom, \exists m' \in Mom \ (m < m' \text{ and } \forall h \in H_{m'}, Choice(\text{Agt}, m)(h) = H_{m'})$

Translation of Coalition Logic to discrete-deterministic STIT

$$
\begin{array}{lcl}
tr(p) & = & \Box p, \text{ for } p \in \text{Prop} \\
tr(\neg\varphi) & = & \neg tr(\varphi) \\
tr(\varphi \vee \psi) & = & tr(\varphi) \vee tr(\psi) \\
tr(\langle\!\![J]\!\!\rangle \varphi) & = & \Diamond [J] \mathbf{X} tr(\varphi)
\end{array}
$$

In STIT terminology

“the coalition J is able to ensure φ”

can be paraphrased by

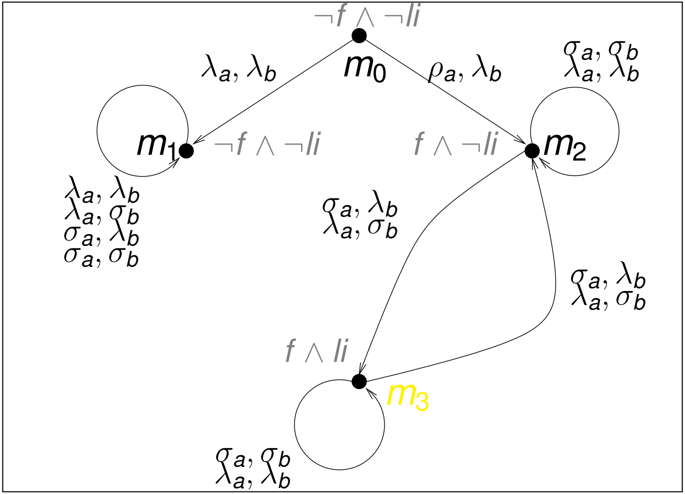
“it is historically possible that J sees to it that next φ”

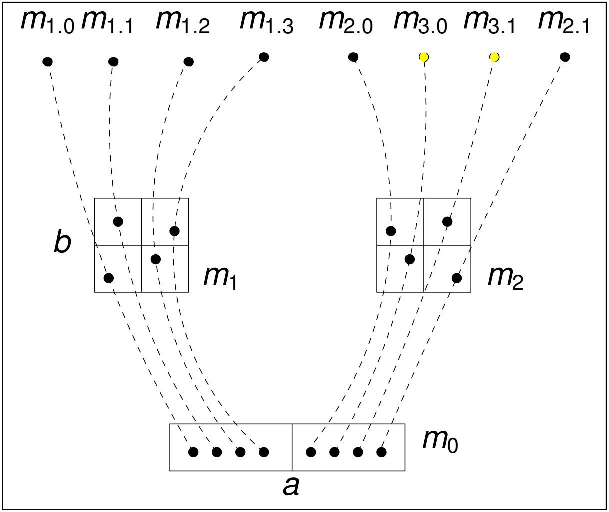
Theorem ([Broersen, Herzig, Troquard 2006])

tr is a correct embedding of CL into discrete-deterministic STIT.

Example: Ann and Bill switch the light

- Four states: m_0, m_1, m_2, m_3
- li = light is on (at m_3)
- f = lamp is functioning (at m_2 and m_3)
- At moment m_0, agent a has the choice between *repairing* a broken lamp (ρ_a) or *remaining passive* (λ_a). Agent b has the vacuous choice of *remaining passive* (λ_b).
- If a chooses not to repair, the system reaches m_1. If a chooses to repair, the system reaches m_2.
- In m_1, m_2 and m_3 both a and b can choose to *toggle* a light switch (τ_a and τ_b) or *not toggle* (λ_a and λ_b).
- If a repairs at m_0 then a and b 'play toggling' between m_2 and m_3


¬f ∧ ¬li
m0
λa, λb
ρa, λb
σa, σb
λa, λb
m1
¬f ∧ ¬li
f ∧ ¬li
m2
λa, λb
λa, σb
σa, λb
σa, σb
σa, λb
λa, σb
σa, λb
λa, σb
f ∧ li
m3
σa, σb
λa, λb



$m_{1.0}$ $m_{1.1}$ $m_{1.2}$ $m_{1.3}$ $m_{2.0}$ $m_{3.0}$ $m_{3.1}$ $m_{2.1}$
b
m_1
m_2
m_0
a

Next:

- Knowing how to play

CL models vs. $BT + AC$ models

Coalition Logic

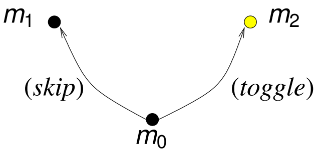
- Neighborhood models
- Game models
- Idea: associate a strategic game (form) to every state

In $BT + AC$ models, indexes

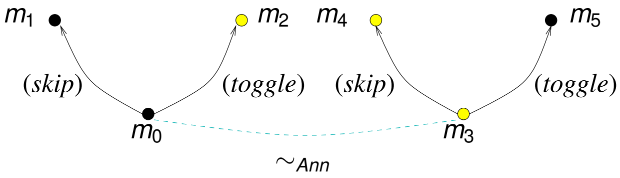
- are ‘part’ of the strategic game,
- and represent
  - the “physical” world, and
  - the current choice/commitment of agents

Helpful modeling power!

- At m_0, the light is off: $m_0 \models \neg li$
- Ann can *toggle* or *skip*
- $m_0 \models \langle\!\![Ann]\!\!\rangle li$
  at m_0, “Ann is able to achieve *li*”



- As before, the light is off: $m_0 \models \neg li$
- Ann is blind and cannot distinguish a world where the light is on from a world where the light is off
- $m_0 \models K_{Ann}\langle\!\langle Ann\rangle\!\rangle li$
  at m_0, "Ann knows she is able to achieve li"



A logical language of action and knowledge must be able to distinguish the following scenarii:

1. the agent *a* knows it has a particular action/choice in its repertoire that ensures φ, possibly without knowing which choice to make to ensure φ.
2. the agent *a* 'knows how to' / 'can' / 'has the power to' ensure φ.

Two readings of "having a strategy"

- $tr(K_J\langle\!\langle J\rangle\!\rangle\varphi) = K_J\Diamond[J]\mathbf{X}\varphi$ **(de dicto)**
  Group J knows (K) there is ($\exists$) a choice s.t. for all ($\forall$) possible outcomes φ
  - Alternating-time *Epistemic* Temporal Logic ATEL [Wooldridge, van der Hoek 2002]
- We want: $\Diamond K_J[J]\mathbf{X}\varphi$ **(de re)**
  There is a choice ($\exists$), s.t. group J knows (K) that for all ($\forall$) possible outcomes φ
  - ATEL does not deal with *de re* strategies [Jamroga 2003], [Schobbens 2004]
  - Several corrections [Schobbens 2004], [Jamroga, van der Hoek 2004], [Jamroga, Ågotnes 2006, 2007]
  - First semantics with STIT [Herzig, Troquard 2006]

Language.

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathbf{X}\varphi \mid [J]\varphi \mid K_i\varphi$$

$BT + AC + K$-models are tuples $\mathcal{M} = (Mom, <, Choice, \sim, V)$ where:

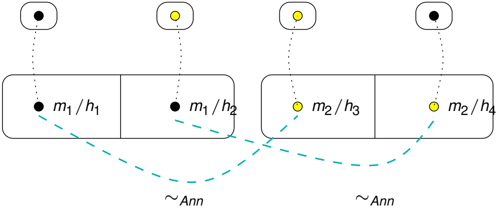
- $(Mom, <, Choice, V)$ is an $BT + AC$-model.
- $\sim \subseteq (Mom \times Hist) \times (Mom \times Hist)$ is a collection of equivalence relations $\sim_i$ (one for every agent $i \in$ Agt) over indexes.

Extra operators:

- $\mathcal{M}, m/h \models K_i\varphi$ iff for all $m'/h' \sim_i m/h$, $\mathcal{M}, m'/h' \models \varphi$

Every K_i is a standard epistemic modality. [Hintikka 1962]

Poor blind Ann again



Epistemic relations are over indexes instead of moments.

- $m_i/h_j \models K_{Ann}\Diamond[Ann]\mathbf{X}\varphi$
  Ann knows she has an action that leads to a lighten moment.
- $m_i/h_j \not\models \Diamond K_{Ann}[Ann]\mathbf{X}\varphi$
  Ann does **not** know how to achieve it.

Outline

Classical logic and naïve specification of actions

Specifying a hammer: if I place a nail (N) and I provide the right force (F), then I can drive a nail (D) with the hammer.

So assume:

$$\vdash N \land F \rightarrow D$$

In classical logic: with one nail and one hammer I can drive in any number of nails I want:

$$\frac{\vdash N \land F \rightarrow D}{\vdash N \land F \rightarrow D \land \ldots \land D}$$

MATHEMATICAL FACTS, NOT RESOURCES

Classical logic:

- Duplicates assumptions (Contraction)

$$\frac{\Gamma, B, B \vdash A}{\Gamma, B \vdash A}\ (\mathrm{C})$$

E.g.: $\vdash p \rightarrow p \wedge p$

- Discards assumptions (Weakening)

$$\frac{\Gamma \vdash A}{\Gamma, B \vdash A}\ (\mathrm{W})$$

E.g.: $\vdash p \wedge q \rightarrow p$

ENGINES, PETROL ENGINES, MIRACULOUS PETROL ENGINES

(From [Girard 1995].)

Consider a petrol engine, in which petrol causes the motion

$$P \vdash M$$

Weakening would enable to call any motion a petrol engine:

$$\frac{\vdash M}{P \vdash M}\ (\mathrm{W})$$

Contraction makes miracles:

$$\dfrac{\dfrac{P \vdash P \qquad P \vdash M}{P, P \vdash P \wedge M}}{P \vdash P \wedge M}\ (\mathrm{C})$$

A fragment of Linear Logic, $\mathcal{L}_{\mathsf{ILL}}$, defined by the BNF

$$A ::= \mathbf{1} \mid p \mid A \otimes A \mid A \& A \mid A \multimap A$$

where $p \in \mathit{Atom}$.

$A \otimes B$: A and B ("composition"; multiplicative conjunction)
$A \& B$: A "and" B ("choice"; additive conjunction)
$A \multimap B$: A implies B ("lollipop"; linear implication)

Let $\bot \in \mathit{Atom}$ a designated atom to mean contradiction.

Negation defined: $\sim A \equiv A \multimap \bot$.

Other connectives in full Linear Logic: $⅋$; $\oplus$; $!$, $?$; $\mathbf{0}$, $\top$.

A fragment of Linear Logic, $\mathcal{L}_{\mathsf{ILL}}$, defined by the BNF

$$A ::= \mathbf{1} \mid p \mid A \otimes A \mid A \& A \mid A \multimap A$$

where $p \in \mathit{Atom}$.

$A \otimes B$: A and B (“composition”; multiplicative conjunction)
$A \& B$: A “and” B (“choice”; additive conjunction)
$A \multimap B$: A implies B (“lollipop”; linear implication)

Let $\bot \in \mathit{Atom}$ a designated atom to mean contradiction.

Negation defined: $\sim A \equiv A \multimap \bot$.

Other connectives in full Linear Logic: $\invamp$; $\oplus$; !, ?; $\mathbf{0}$, $\top$.

Γ and Γ' are finite multisets of formulas. (Exchange rule holds implicitly.)

$$\frac{}{A \vdash A}\ \text{ax} \qquad \frac{\Gamma, A \vdash C \qquad \Gamma' \vdash A}{\Gamma, \Gamma' \vdash C}\ \text{cut}$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}\ \otimes\text{L} \qquad \frac{\Gamma \vdash A \qquad \Gamma' \vdash B}{\Gamma, \Gamma' \vdash A \otimes B}\ \otimes\text{R} \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \& B}$$

$$\frac{\Gamma \vdash A \qquad \Gamma', B \vdash C}{\Gamma', \Gamma, A \multimap B \vdash C}\ \multimap\text{L} \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}\ \multimap\text{R}$$

$$\frac{\Gamma \vdash C}{\Gamma, \mathbf{1} \vdash C}\ \mathbf{1}\text{L} \qquad \frac{}{\vdash \mathbf{1}}\ \mathbf{1}\text{R}$$

Γ and Γ' are finite multisets of formulas. (Exchange rule holds implicitly.)

$$\frac{}{A \vdash A}\,\text{ax} \qquad \frac{\Gamma, A \vdash C \qquad \Gamma' \vdash A}{\Gamma, \Gamma' \vdash C}\,\text{cut}$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}\,\otimes\text{L} \qquad \frac{\Gamma \vdash A \qquad \Gamma' \vdash B}{\Gamma, \Gamma' \vdash A \otimes B}\,\otimes\text{R} \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \& B}$$

$$\frac{\Gamma \vdash A \qquad \Gamma', B \vdash C}{\Gamma', \Gamma, A \multimap B \vdash C}\,\multimap\text{L} \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}\,\multimap\text{R}$$

$$\frac{\Gamma \vdash C}{\Gamma, \mathbf{1} \vdash C}\,\mathbf{1}\text{L} \qquad \frac{}{\vdash \mathbf{1}}\,\mathbf{1}\text{R}$$

HILBERT SYSTEM $\vdash_{\mathsf{H}}$

- $A \multimap A$
- $(A \multimap B) \multimap ((B \multimap C) \multimap (A \multimap C))$
- $(A \multimap (B \multimap C)) \multimap (B \multimap (A \multimap C))$
- $A \multimap (B \multimap A \otimes B)$
- $(A \multimap (B \multimap C)) \multimap (A \otimes B \multimap C)$
- $\mathbf{1}$
- $\mathbf{1} \multimap (A \multimap A)$

$\multimap$-rule: if $\vdash_{\mathsf{H}} A$, $\vdash_{\mathsf{H}} A \multimap B$ then $\vdash_{\mathsf{H}} B$

Outline

Classically, E_aA reads "agent a brings about A".
Here, E_aA reads "agent a brings about the resource A."

Principles:

1. If two statements are equivalent, then bringing about one is equivalent to bringing about the other.

$$\frac{A \vdash B \qquad B \vdash A}{E_aA \vdash E_aB}\ E_a(\text{re})$$

2. If something is brought about, then this something holds.

$$\frac{\Gamma, A \vdash B}{\Gamma, E_aA \vdash B}\ \text{act}(a)$$

3. It is not possible to bring about a tautology.

$$\frac{\vdash A}{E_aA \vdash \bot}\ {\sim}\text{nec}$$

- A semantics (instantiation of modal resource Kripke models);
- The calculus is sound and complete;
- The *cut* rule can be eliminated;
- Proof search is in PSPACE.
- See [Porello & Troquard 2014], [Porello & Troquard 2015]

An electric screwdriver has two components:

- A power-pistol (p) produces some rotational force (F) when the button is pushed (P): $E_p(P \multimap F)$.
- The screwdriver bit (b) tightens a loose screw (S) when a rotational force (F) is applied: $E_b(S \otimes F \multimap T)$.

VERY SIMPLE CASE OF PERSON-ARTEFACT INTERACTION

Suppose:

- we have an electric screwdriver (p and b);
- we have a loose screw (S);
- agent a pushes the button of the pistol ($E_a P$).

So

- we can have a tighten screw (T):

$$E_b(S \otimes F \multimap T), E_p(P \multimap F), E_a P, S \vdash T$$

- we cannot have two tighten screws:

$$E_b(S \otimes F \multimap T), E_p(P \multimap F), E_a P, S \nvdash T \otimes T$$

AN AUTOMATICALLY GENERATED PLAN[1]

```
prove> [b](S*F->T),[p](P->F),[a]P,S ==> T

     ------- Ax
     P ==> P
     ----------  act([a])  ------- Ax
     [a]P ==> P            F ==> F
     ------------------------------ L->
             P->F, [a]P ==> F
------- Ax   ----------------------  act([p])
S ==> S      [p](P->F), [a]P ==> F
----------------------------------- R*   ------- Ax
    [p](P->F), [a]P, S ==> S*F           T ==> T
    ---------------------------------------------- L->
        S*F->T, [p](P->F), [a]P, S ==> T
        ------------------------------------------ act([b]
        [b](S*F->T), [p](P->F), [a]P, S ==> T
```

[1] http://www.loa.istc.cnr.it/personal/troquard/SOFTWARES/MLLPROVER/mllprover.html

Modelling abilities (I)

There is a neat difference between classical and resource-sensitive reasoning. Suppose a theory where:

$$\frac{}{A \vdash B}\ \text{ax1} \qquad \frac{}{A \vdash C}\ \text{ax2}$$

Imagine a situation where A, and 'exactly' A.

Classical logic:

- can have B, can have C, can have $B \wedge C$.
- do have B, C, $B \wedge C$, and A.

Linear Logic:

- can have B, can have C.
- do not have either of B or C before making a choice of what rule to apply.
- cannot have $B \otimes C$.

Modelling abilities (II)

Interpreting $E_a A$ in Linear Logic:

- Left of a sequent: an actual resource of a bringing about A.
- Right of a sequent: an ability of a to bring about A by consuming some resources.

Consider now coalitions too: $E_C A$ with C a set of agents.[2]

For instance:

$$\frac{\Gamma, E_C(A \otimes B) \vdash D}{\Gamma, E_C A, E_C B \vdash D} \quad \text{but} \quad \frac{\Gamma \vdash E_{C_1} A \qquad \Delta \vdash E_{C_2} B}{\Gamma, \Delta \vdash E_{C_1 \cup C_2}(A \otimes B)}\ , C_1 \cap C_2 = \emptyset$$

and maybe

$$\frac{\Gamma \vdash E_{C_1} A \qquad \Gamma \vdash E_{C_2} B}{\Gamma \vdash E_{C_1 \cup C_2}(A \& B)}$$

[2] See prototype implementation:
http://www.loa.istc.cnr.it/personal/troquard/SOFTWARES/MLLPROVER/mllprover.html

M. R. Ayers.
The refutation of determinism.
Methuen, 1968.

Jan Broersen, Andreas Herzig, and Nicolas Troquard.
From Coalition Logic to STIT.
Electr. Notes Theor. Comput. Sci., 157(4):23–35, 2006.

Jan Broersen, Andreas Herzig, and Nicolas Troquard.
A normal simulation of coalition logic and an epistemic extension.
In *Proceedings of the 11th Conference on Theoretical Aspects of Rationality and Knowledge (TARK-2007), Brussels, Belgium, June 25-27, 2007*, pages 92–101, 2007.

Jan Broersen, Andreas Herzig, and Nicolas Troquard.
What groups do, can do, and know they can do: an analysis in normal modal logics.
Journal of Applied Non-Classical Logics, 19(3):261–290, 2009.

Dag Elgesem.
Action theory and modal logic.
PhD thesis, Universitetet i Oslo, 1993.

Dag Elgesem.
The modal logic of agency.
Nordic J. Philos. Logic, 2(2):1–46, 1997.

Marcelo Finger and Dov M. Gabbay.
Adding a temporal dimension to a logic system.
Journal of Logic, Language and Information, 1:203–233, 1992.

REFERENCES II

Jean-Yves Girard.
Linear logic.
Theoretical Computer Science, 50(1):1–101, 1987.

Valentin Goranko, Wojciech Jamroga, and Paolo Turrini.
Strategic games and truly playable effectivity functions.
Autonomous Agents and Multi-Agent Systems, 26(2):288–314, 2013.

Andreas Herzig and Nicolas Troquard.
Knowing how to play: uniform choices in logics of agency.
In *5th International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS 2006), Hakodate, Japan, May 8-12, 2006*, pages 209–216, 2006.

D. Hume.
A Treatise of Human Nature.
Oxford University Press, 1888.
L. A. Selby-Bigge (ed.).

Wojciech Jamroga and Thomas Ågotnes.
Constructive knowledge: what agents can achieve under imperfect information.
Journal of Applied Non-Classical Logics, 17(4):423–475, 2007.

Wojciech Jamroga and Wiebe van der Hoek.
Agents that know how to play.
Fundam. Inform., 63(2-3):185–219, 2004.

A. Kenny.
Will, Freedom and Power.
Blackwell, 1975.

Alfred R. Mele.
Agent's Abilities.
Noûs, pages 447–470, 2003.

Peter Morriss.
Power: A Philosophical Analysis.
manchester University Press, 1 edition, 1987.

Marc Pauly.
A modal logic for coalitional power in games.
J. Log. Comput., 12(1):149–166, 2002.

Daniele Porello and Nicolas Troquard.
A resource-sensitive logic of agency.
In *ECAI 2014 - 21st European Conference on Artificial Intelligence*, pages 723–728, 2014.

Daniele Porello and Nicolas Troquard.
Non-normal modalities in variants of linear logic.
Journal of Applied Non-Classical Logics, 25(3):229–255, 2015.

Pierre-Yves Schobbens.
Alternating-time logic with imperfect recall.
Electr. Notes Theor. Comput. Sci., 85(2):82–93, 2004.

F. Santos, A. Jones, and J. Carmo.
Responsibility for Action in Organisations: a Formal Model.
In G. Holmström-Hintikka and R. Tuomela, editors, *Contemporary Action Theory*, volume 1, pages 333–348. Kluwer, 1997.

REFERENCES IV

N. Troquard.
Reasoning about coalitional agency and ability in the logics of “bringing-it-about”.
Autonomous Agents and Multi-Agent Systems, 28(3):381–407, 2014.

Wiebe van der Hoek and Michael Wooldridge.
Tractable multiagent planning for epistemic goals.
In *Proc. First International Joint Conference on Autonomous Agents & Multiagent Systems, AAMAS 2002*, pages 1167–1174, 2002.