The Distributed Ontology, Model and Specification Language (DOL)
Day 4: Semantics of Structured OMS

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Summary of Day 3

On Day 3 we have looked at:

- Assembling OMS from pieces:
  Basic OMS, union, translation
- Making a large OMS smaller:
  module extraction, approximation, reduction, filtering
- Non-monotonic reasoning through employing
  a closed-world assumption:
  minimization, maximization, freeness, cofreeness

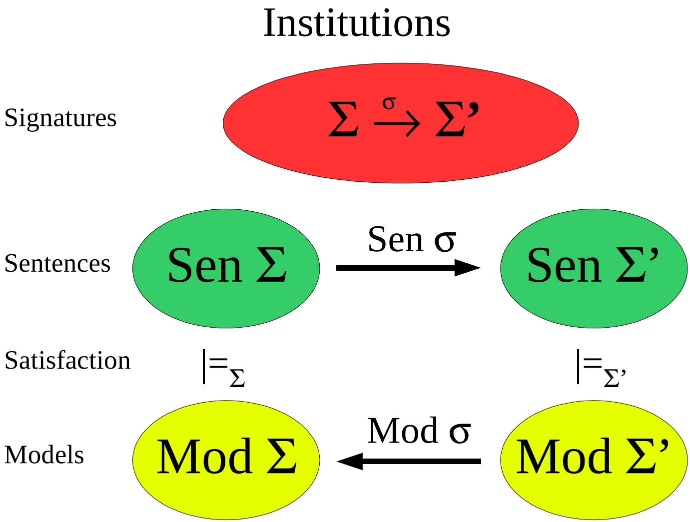
Today

We will focus today on:

- Semantics of structured OMS
  - based on institutions
- Proofs in OMS
  - based on entailment systems

Semantics of OMS

Institutions (intuition)



Some Basic Category Theory

Our use of category theory is modest, oriented towards providing easy proofs for very general results.

Definition (Category)

A category **C** is a graph together with a partial composition operation defined on edges that match:

if $f: A \rightarrow B$ and $g: B \rightarrow C$, then $f;g : A \rightarrow C$.

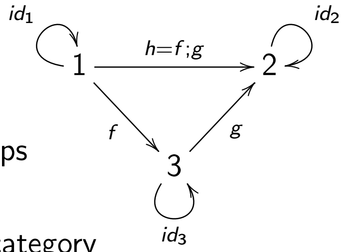
Graph nodes are called objects, graph edges are called morphisms.

Requirements on a category: morphisms behave monoid-like, that is,

- Composition has a neutral element $id_A: A \rightarrow A$ (for each object $A \in |\mathbf{C}|$):
  for $f : A \rightarrow B$, $id_A; f = f$ and $f; id_B = f$
- Composition is associative:
  $(f;g);h = f;(g;h)$ if both sides are defined

Categories: Examples

- sets and functions
- FOL signatures and signature morphisms
- OWL signatures and signature morphisms
- logical theories and theory morphisms
- groups and group homomorphisms
- general algebras and homomorphisms
- metric spaces and contractions
- topological spaces and continuous maps
- automata and simulations
- each pre-order, seen as a graph, is a category
- each monoid is a category with one object



Opposite Categories

Definition (Opposite category)

Given a category **C**, its opposite category $\mathbf{C}^{op}$ has the same objects and morphism as **C**, but with all morphisms reversed. That is,

$$\text{if } f\colon A \to B \in \mathbf{C}, \text{ then } f\colon B \to A \in \mathbf{C}^{op}.$$

$$\text{if } f; g = h \text{ in } \mathbf{C}, \text{ then } g; f = h \text{ in } \mathbf{C}^{op}.$$

Functors

Definition (Functor)

Given categories $\mathbf{C}_1$ and $\mathbf{C}_2$, a functor $F\colon \mathbf{C}_1 \to \mathbf{C}_2$ is a graph homomorphism $F\colon \mathbf{C}_1 \to \mathbf{C}_2$ preserving the monoid structure, that is

- Neutral elements are preserved:

  $$F(id_A) = id_{F(A)}$$

  for each object $A \in |\mathbf{C}|$
- Composition is preserved:

  $$F(f;g) = F(f);F(g)$$

  for each $f\colon A \to B$, $g\colon B \to C \in \mathbf{C}$.

Institutions (formal definition)

An institution $\mathcal{I} = \langle \mathbf{Sign}, \mathbf{Sen}, \mathbf{Mod}, \langle \models_\Sigma \rangle_{\Sigma \in |\mathbf{Sign}|} \rangle$ consists of:

- a category **Sign** of signatures;
- a functor $\mathbf{Sen} \colon \mathbf{Sign} \to \mathbf{Set}$, giving a set $\mathbf{Sen}(\Sigma)$ of Σ-sentences for each signature $\Sigma \in |\mathbf{Sign}|$, and a function $\mathbf{Sen}(\sigma) \colon \mathbf{Sen}(\Sigma) \to \mathbf{Sen}(\Sigma')$ that yields σ-translation of Σ-sentences to Σ'-sentences for each $\sigma \colon \Sigma \to \Sigma'$;
- a functor $\mathbf{Mod} \colon \mathbf{Sign}^{op} \to \mathbf{Cat}$, giving a category $\mathbf{Mod}(\Sigma)$ of Σ-models for each signature $\Sigma \in |\mathbf{Sign}|$, and a functor $\_|_\sigma = \mathbf{Mod}(\sigma) \colon \mathbf{Mod}(\Sigma') \to \mathbf{Mod}(\Sigma)$; for each $\sigma \colon \Sigma \to \Sigma'$;
- for each $\Sigma \in |\mathbf{Sign}|$, a satisfaction relation $\models_{\mathcal{I},\Sigma} \subseteq \mathbf{Mod}(\Sigma) \times \mathbf{Sen}(\Sigma)$

such that for any signature morphism $\sigma \colon \Sigma \to \Sigma'$, Σ-sentence $\varphi \in \mathbf{Sen}(\Sigma)$ and Σ'-model $M' \in \mathbf{Mod}(\Sigma')$:

$$M' \models_{\mathcal{I},\Sigma'} \sigma(\varphi) \text{ iff } M'|_\sigma \models_{\mathcal{I},\Sigma} \varphi \qquad \text{[Satisfaction condition]}$$

Sample Institutions

- Prop, FOL and OWL are institutions
  we have proven the satisfaction conditions in lecture 2

Plenty of Institutions

- Lary Moss' logics from his ESSLLI evening talk on Tuesday
- first-order, higher-order logic, polymorphic logics
- logics of partial functions
- modal logic (epistemic logic, deontic logic, description logics, logics of knowledge and belief, agent logics)
- μ-calculus, dynamic logic
- spatial logics, temporal logics, process logics, object logics
- intuitionistic logic
- linear logic, non-monotonic logics, fuzzy logics
- paraconsistent logic, database query languages

Working in an Arbitrary Logical System

Many notions and results generalise to an arbitrary institution:

- logical consequence
- logical theory
- satisfiability
- conservative extension
- theory morphism
- many more . . .

In the sequel, fix an arbitrary instution I.

Weakly inclusive institutions

Definition (adopted from Goguen, Roșu)

A weakly inclusive category is a category having a singled out class of morphisms (called inclusions) which is closed under identities and composition. Inclusions hence form a partial order.
An weakly inclusive institution is one with an inclusive signature category such that

- the sentence functor preserves inclusions,
- the inclusion order has a least element (denote $\emptyset$), suprema (denoted $\cup$), infima (denoted $\cap$), and differences (denoted $\setminus$),
- model categories are weakly inclusive.

$M|_\Sigma$ means $M|_\iota$ where $\iota : \Sigma \to Sig(M)$ is the inclusion.
In the sequel, fix an arbitrary weakly inclusive instution I.

Semantic domains for OMS in DOL

Flattenable OMS (can be flattened to a basic OMS)

- basic OMS
- extensions, unions, translations
- approximations, module extractions, filterings (flattenable)
- combinations of networks (flattenable)
- semantics: (Σ, Ψ) (theory-level)
  - Σ: a signature in I, also written $Sig(O)$
  - Ψ: a set of Σ-sentences, also written $Th(O)$

Elusive OMS (= non-flattenable OMS)

- reductions, minimization, maximization, (co)freeness (elusive)
- semantics: $(\Sigma, \mathcal{M})$ (model-level)
  - Σ: a signature in I, also written $Sig(O)$
  - $\mathcal{M}$: a class of Σ-models, also written $Mod(O)$

We can obtain the model-level semantics from the theory-level semantics by taking $\mathcal{M} = \{M \in \mathbf{Mod}(\Sigma) \mid M \models \Psi\}$.

Semantics of basic OMS

We assume that $[\![O]\!]_{basic} = (\Sigma, \Psi)$ for some OMS language based on I. The semantics consists of

- a signature Σ in I
- a set Ψ of Σ-sentences

This direct leads to a theory-level semantics for OMSx:

$$[\![O]\!]_\Gamma^T = [\![O]\!]_{basic}$$

Generally, if a theory-level semantics is given: $[\![O]\!]_\Gamma^T = (\Sigma, \Psi)$, this leads to a model-level semantics as well:

$$[\![O]\!]_\Gamma^M = (\Sigma, \{M \in Mod(\Sigma) \mid M \models \Psi\})$$

Semantics of extensions

O_1 flattenable $[\![O_1 \textbf{ then } O_2]\!]^T_\Gamma = (\Sigma_1 \cup \Sigma_2, \Psi_1 \cup \Psi_2)$

where

- $[\![O_1]\!]^T_\Gamma = (\Sigma_1, \Psi_1)$
- $[\![O_2]\!]_{basic} = (\Sigma_2, \Psi_2)$

O_1 elusive $[\![O_1 \textbf{ then } O_2]\!]^M_\Gamma = (\Sigma_1 \cup \Sigma_2, \mathcal{M}')$

where

- $[\![O_1]\!]^M_\Gamma = (\Sigma_1, \mathcal{M}_1)$
- $[\![O_2]\!]_{basic} = (\Sigma_2, \Psi_2)$
- $\mathcal{M}' = \{M \in \textbf{Mod}(\Sigma_1 \cup \Sigma_2) \mid M \models \Psi_2, M|_{\Sigma_1} \in \mathcal{M}_1\}$

Semantics of extensions (cont'd)

%mcons (%def, %mono) leads to the additional requirement that

each model in $\mathcal{M}_1$ has a (unique, unique up to isomorphism) $\Sigma_1 \cup \Sigma_2$-expansion to a model in $\mathcal{M}'$.

%implies leads to the additional requirements that

$\Sigma_2 \subseteq \Sigma_1$ *and* $\mathcal{M}' = \mathcal{M}_1$.

%ccons leads to the additional requirement that

$\mathcal{M}' \models \varphi$ *implies* $\mathcal{M}_1 \models \varphi$ *for any* Σ_1*-sentence* φ.

Theorem

%mcons implies %ccons, but not vice versa.

References to Named OMS

- Reference to an OMS existing on the Web
- written directly as a URL (or IRI)
- Prefixing may be used for abbreviation

```
http://owl.cs.manchester.ac.uk/co-ode-files/
ontologies/pizza.owl
```

```
co-ode:pizza.owl
```

Semantics Reference to Named OMS: $[\![iri]\!]_\Gamma = \Gamma(iri)$
where Γ is a global map of IRIs to OMS denotations

Semantics of unions

O_1, O_2 flattenable $[\![O_1 \text{ and } O_2]\!]^T_\Gamma = (\Sigma_1 \cup \Sigma_2, \Psi_1 \cup \Psi_2)$, where

- $[\![O_i]\!]^T_\Gamma = (\Sigma_i, \Psi_i)$ $(i = 1, 2)$

one of O_1, O_2 elusive $[\![O_1 \text{ and } O_2]\!]^M_\Gamma = (\Sigma_1 \cup \Sigma_2, \mathcal{M})$, where

- $[\![O_i]\!]^M_\Gamma = (\Sigma_i, \mathcal{M}_i)$ $(i = 1, 2)$
- $\mathcal{M} = \{M \in \mathbf{Mod}(\Sigma_1 \cup \Sigma_2) \mid M|_{\Sigma_i} \in \mathcal{M}_i, i = 1, 2\}$

Semantics of translations

O flattenable Let $[\![O]\!]^T_\Gamma = (\Sigma, \Psi)$. Then

$$[\![O \textbf{ with } \sigma : \Sigma \to \Sigma']\!]^T_\Gamma = (\Sigma', \sigma(\Psi))$$

O elusive Let $[\![O]\!]^M_\Gamma = (\Sigma, \mathcal{M})$. Then

$$[\![O \textbf{ with } \sigma : \Sigma \to \Sigma']\!]^M_\Gamma = (\Sigma', \mathcal{M}')$$

where $\mathcal{M}' = \{M \in \mathbf{Mod}(\Sigma') \mid M|_\sigma \in \mathcal{M}\}$

Hide – Extract – Forget – Select

	hide/reveal	remove/extract	forget/keep	select/reject
semantic background	model reduct	conservative extension	uniform interpolation	theory filtering
relation to original	interpretable	subtheory	interpretable	subtheory
approach	model level	theory level	theory level	theory level
type of OMS	elusive	flattenable	flattenable	flattenable
signature of result	$= \Sigma$	$\geq \Sigma$	$= \Sigma$	$\geq \Sigma$
change of logic	possible	not possible	possible	not possible
application	specification	ontologies	ontologies	blending

Semantics of reductions

Let $[\![O]\!]_\Gamma^M = (\Sigma, \mathcal{M})$

- $[\![O$ **reveal** $\Sigma']\!]_\Gamma^M = (\Sigma', \mathcal{M}|_{\Sigma'})$, where $\mathcal{M}|_{\Sigma'} = \{M|_{\Sigma'} \mid M \in \mathcal{M}\})$
- $[\![O$ **hide** $\Sigma']\!]_\Gamma^M = [\![O$ **reveal** $\Sigma \setminus \Sigma']\!]_\Gamma^M$

$\mathcal{M}|_{\Sigma'}$ may be impossible to capture by a theory (even if $\mathcal{M}$ is).

Modules

Definition

$O' \subseteq O$ is a Σ-module of (flat) O iff O is a model-theoretic Σ-conservative extension of O', i.e. for every model M of O', $M|_\Sigma$ can be expanded to an O-model.

Depleting modules

Definition

Let O_1 and O_2 be two OMS and $\Sigma \subseteq Sig(O_i)$.
Then O_1 and O_2 are Σ-inseparable ($O_1 \equiv_\Sigma O_2$) iff

$$Mod(O_1)|_\Sigma = Mod(O_2)|_\Sigma$$

Definition

$O' \subseteq O$ is a depleting Σ-module of (flat) O iff $O \setminus O' \equiv_{\Sigma \cup Sig(O')} \emptyset$.

Theorem

1. *Depleting Σ-modules are Σ-conservative.*
2. *The minimum depleting Σ-module always exists.*

Semantics of module extraction (remove/extract)

Note: O must be flattenable!

Let $[\![O]\!]_\Gamma^T = (\Sigma, \Psi)$.
$[\![O \textbf{ extract } \Sigma_1]\!]_\Gamma^T = (\Sigma_2, \Psi_2)$
where $(\Sigma_2, \Psi_2) \subseteq (\Sigma, \Psi)$ is the minimum depleting Σ_1-module of (Σ, Ψ)

$[\![O \textbf{ remove } \Sigma_1]\!]_\Gamma^T = [\![O \textbf{ extract } \Sigma \setminus \Sigma_1]\!]_\Gamma^T$

Tools can extract other types of module though (i.e. using locality). However, any two modules will have the same Σ-consequences.

Semantics of interpolation (forget/keep)

Note: O must be flattenable!
Let $[\![O]\!]^T_\Gamma = (\Sigma, \Psi)$.

$[\![O \textbf{ keep in } \Sigma']\!]^T_\Gamma = (\Sigma', \{\varphi \in \mathbf{Sen}(\Sigma') \mid \Psi \models \varphi\})$
Note: any logically equivalent theory will also do).
Challenge: find a finite theory (= uniform interpolant). This is not always possible, and sometimes theoretically possible but not computable.

$[\![O \textbf{ forget } \Sigma']\!]^T_\Gamma = [\![O \textbf{ keep in } \Sigma \setminus \Sigma']\!]^T_\Gamma$

Semantics of select/reject

Note: O must be flattenable!

Let $[\![O]\!]^T_\Gamma = (\Sigma, \Psi)$.

$[\![O \textbf{ select } (\Sigma', \Phi)]\!]^T_\Gamma = (\Sigma, Sen(\iota)^{-1}(\Psi) \cup \Phi)$
where $\iota : \Sigma' \to \Sigma$ is the inclusion

$[\![O \textbf{ reject } (\Sigma', \Phi)]\!]^T_\Gamma = (\Sigma \setminus \Sigma', Sen(\iota)^{-1}(\Psi) \setminus \Phi)$
where $\iota : \Sigma \setminus \Sigma' \to \Sigma$ is the inclusion

Relations among the different notions

$$
\begin{array}{rl}
 & \mathit{Mod}(O \ \textbf{reveal} \ \Sigma) \\
= & \mathit{Mod}(O \ \textbf{extract} \ \Sigma)|_{\mathit{Sig}(O) \setminus \Sigma} \\
\subseteq & \mathit{Mod}(O \ \textbf{keep} \ \Sigma) \\
\subseteq & \mathit{Mod}(O \ \textbf{select} \ \Sigma)
\end{array}
$$

Semantics of minimizations

Let $[\![O_1]\!]^M_\Gamma = (\Sigma_1, \mathcal{M}_1)$
Let $[\![O_1 \textbf{ then } O_2]\!]^M_\Gamma = (\Sigma_2, \mathcal{M}_2)$
Then

$$[\![O_1 \textbf{ then minimize } O_2]\!]^M_\Gamma = (\Sigma_2, \mathcal{M})$$

where
$\mathcal{M} = \{M \in \mathcal{M}_2 \mid M \text{ is minimal in } \{M' \in \mathcal{M}_2 \mid M'|_{\Sigma_1} = M|_{\Sigma_1}\}\}$

Note that in a weakly inclusive institution, inclusion model morphisms provide a partial order on models.

Dually: maximization.

Initial Objects

Definition

An object I in a category $\mathbf{C}$ is called an initial object, if for each object $A \in |\mathbf{C}|$, there is a unique morphism $I \rightarrow A$.

Example

Inititial objects in different categories:

- sets and functions: the empty set
- FOL signatures: the empty signature
- algebras and homomorphisms: the term algebra
- models of Horn clauses: the Herbrand model

Theorem

Initial objects are unique up to isomorphism.

Semantics of freeness

We only treat the special case of **free** $\{O\}$.

Let $[\![O]\!]_\Gamma^M = (\Sigma, \mathcal{M})$ Then

$$[\![\textbf{free}\ O]\!]_\Gamma^M = (\Sigma, \{M \in \mathcal{M} \mid M \text{ is initial in } \mathcal{M}\})$$

Semantics of interpretations

Let $[\![O_i]\!]^M_\Gamma = (\Sigma_i, \mathcal{M}_i)$ $(i = 1, 2)$

$$[\![\textbf{interpretation } \mathit{IRI} : O_1 \textbf{ to } O_2 = \sigma]\!]^M_\Gamma$$

is defined iff

$$\mathit{Mod}(\sigma)(\mathcal{M}_2) \subseteq \mathcal{M}_1$$

Note that this is the same condition as for theory morphisms.

Proof calculus

Logical Consequences and Refinement of OMS

Definition (Logical Consequences of an OMS)

$$O \models_\Sigma \varphi \qquad \text{iff} \qquad \Sigma = Sig(O),\ M \models_\Sigma \varphi \text{ for all } M \in Mod(O)$$

Definition (Refinement between two OMS)

$$O \rightsquigarrow O' \qquad \text{iff} \qquad Mod(O') \subseteq Mod(O)$$

Entailment systems

Definition

Given an institution $\mathcal{I} = (\mathbf{Sign}, \mathbf{Sen}, Mod, \models)$, an entailment system $\vdash$ for $\mathcal{I}$ consists of relations $\vdash_\Sigma \subseteq \mathcal{P}(\mathbf{Sen}(\Sigma)) \times \mathbf{Sen}(\Sigma)$ such that

1. reflexivity: for any $\varphi \in \mathbf{Sen}(\Sigma)$, $\{\varphi\} \vdash_\Sigma \varphi$,
2. monotonicity: if $\Gamma \vdash_\Sigma \varphi$ and $\Gamma' \supseteq \Gamma$ then $\Gamma' \vdash_\Sigma \varphi$,
3. transitivity: if $\Gamma \vdash_\Sigma \varphi_i$ for $i \in I$ and $\Gamma \cup \{\varphi_i \mid i \in I\} \vdash_\Sigma \psi$, then $\Gamma \vdash_\Sigma \psi$,
4. $\vdash$-translation: if $\Gamma \vdash_\Sigma \varphi$, then for any $\sigma : \Sigma \longrightarrow \Sigma'$ in $\mathbf{Sign}$, $\sigma(\Gamma) \vdash_{\Sigma'} \sigma(\varphi)$,
5. soundness: if $\Gamma \vdash_\Sigma \varphi$ then $\Gamma \models_\Sigma \varphi$.

The entailment system is complete if, in addition, $\Gamma \models_\Sigma \varphi$ implies $\Gamma \vdash_\Sigma \varphi$.

Proof calculus for entailment (Borzyszkowski) covering some part of DOL

$$(CR)\ \frac{\{O \vdash \varphi_i\}_{i \in I} \quad \{\varphi_i\}_{i \in I} \vdash \varphi}{O \vdash \varphi} \qquad (basic)\ \frac{\varphi \in \Gamma}{\langle \Sigma, \Gamma \rangle \vdash \varphi}$$

$$(sum1)\ \frac{O_1 \vdash \varphi}{O_1 \ \textbf{and} \ O_2 \vdash \varphi} \qquad (sum2)\ \frac{O_2 \vdash \varphi}{O_1 \ \textbf{and} \ O_2 \vdash \varphi}$$

$$(trans)\ \frac{O \vdash \varphi}{O \ \textbf{with} \ \sigma \vdash \sigma(\varphi)} \qquad (derive)\ \frac{O \vdash \sigma(\varphi)}{O \ \textbf{hide} \ \sigma \vdash \varphi}$$

Soundness means: $O \vdash \varphi$ implies $O \models \varphi$

Completeness means: $O \models \varphi$ implies $O \vdash \varphi$

Proof calculus for refinement (Borzyszkowski)

$$(\mathit{Basic})\ \frac{O \vdash \Gamma}{\langle \Sigma, \Gamma \rangle \rightsquigarrow O} \qquad (\mathit{Sum})\ \frac{O_1 \rightsquigarrow O \quad O_2 \rightsquigarrow O}{O_1 \textbf{ and } O_2 \rightsquigarrow O}$$

$$(\mathit{Trans})\ \frac{O \rightsquigarrow O' \textbf{ hide } \sigma}{O \textbf{ with } \sigma \rightsquigarrow O'}$$

$$(\mathit{Derive})\ \frac{O \rightsquigarrow O''}{O \textbf{ hide } \sigma \rightsquigarrow O'}$$

if $\sigma : O' \longrightarrow O''$
is a conservative extension

Soundness means: $O_1 \rightsquigarrow O_2$ implies $O_1 \rightsquigarrow O_2$
Completeness means: $O_1 \rightsquigarrow O_2$ implies $O_1 \rightsquigarrow O_2$

Soundness and Completeness

Theorem (Borzyszkowski, Tarlecki, Diaconescu)

The calculi for structured entailment and refinement are sound.
Under the assumptions that

- *the institution admits Craig-Robinson interpolation,*
- *the institution has weak model amalgamation, and*
- *the entailment system is complete,*

the calculi are also complete.

For refinement, we need an oracle for conservative extensions.
Craig-Robinson interpolation, weak model amalgamation:
technical model-theoretic conditions