Countability in the Nominal and Verbal Domains

August 15, 2016

ESSLLI 2016, Bolzano

Advanced Course

Hana Filip & Peter Sutton

hana.filip@gmail.com

peter.r.sutton@icloud.com

Department of Linguistics

Heinrich-Heine-University Düsseldorf

Course outline

Day 1: Introduction to mereological semantics and the mass/count distinction (Link, Krifka)
Day 2: Chierchia: Counting is counting 'stable' atoms
Rothstein: Counting is counting 'semantic atoms'
Day 3: Landman: Counting is counting non-overlapping 'generators'
Sutton & Filip: Counting based on a multi-feature theory of individuation
Day 4: Mereological event semantics
Day 5: Counting and measuring in the verbal domain
Filip 2008

1 Main data: What is to be explained?

1.1 The mass/count distinction in English: Morphological and syntactic criteria

- Count Ns: have plural forms and direct modifications with numericals.
- Mass Ns: do not have plural forms and direct modifications with numericals are ungrammatical or highly marked, unless they can be coerced into a count interpretation.

	[–C]		[+C]	
Plural marking	salt furniture	#salts #furnitures	boy	✓boys
Numericals	#one salt #one furniture	#two salt(s) #two furniture(s)	✓one boy	✓two boys
NP	Salt was sprinkled over the floor.		*Lentil was sprinkled over the floor. (Lentils were sprinkled over the floor.)	

Coercion:

- Mass to count in count syntax *three ice-creams*: (i) portions (e.g., cones); (ii) subkinds in the taxonomic interpretation (e.g., vanilla, lime, strawberry).
- Count-to-mass in mass syntax: *There was lentil in the curry, so I'm sure that lentil made the magic in here* reference to the stuff lentils are made of.

1.2 The mass/count distinction: Cross-linguistic perspective

- Most languages have some way of marking the mass/count distinction, but there is a substantial variation in the ways in which it is marked across different languages.

- Number marking (plural morphology) may not be required even in languages which have a grammaticized lexical singular/plural distinction. Turkish: the singular is used with any number (Krifka 1995, p.407):

(1) a. üç elma — three apple.SG b. * üç elma-lar — three apple-PL TURKISH

- In Classifier languages, lassifiers are required in modifications with numericals. MANDARIN
  Nouns select for different classifiers depending on their countability status.
  gè: with nouns denoting individual things or people, but not stuff like mud

(2) a. wǔ gè píng guǒ — five CL apple — 'five apples'
b. * wǔ píng guǒ — five apple — 'five apples' (intended)
c. *wǔ gè nì — five CL mud — 'five [quantities] of mud (intend.)

- Case agreement is required in felicitous modifications with numericals

(3) a. kolme kissa-a — three cat.SG-PART b. # kolme riisi-a — three rice.SG-PART FINNISH

(4) Some languages, such as Native American languages, including Hopi (Whorf 1939), Lillooet Salish (Davis and Matthewson 1999), Halkomelem Salish (Wiltschko 2007), do not seem to have a grammaticized mass/count distinction among nouns.

1.3 Cognitive criteria

- **MASS : COUNT = STUFF : OBJECT**

  The linguistic mass/count distinction to a certain extent reflects the fundamental conceptual and pre-linguistic distinction between 'stuff' and 'object', and its grounding in the structure of the matter in the world. Some examples:

  – Basic words for undifferentiated stuff like blood or air are encoded as mass in natural languages (potential substantive universal, see Chierchia 2010, p.105)

  – Conceptual pre-linguistic distinction between stuff and individuated objects bootstraps the acquisition of the mass/count distinction (Macnamara 1982; Spelke et al 1985; Soja, Carey and Spelke 1991, i.e., contra Quine 1960):

    Basic words for "Spelke" objects (Spelke et al 1985) tend to be encoded as count.

    "Spelke" objects are individuated objects with discrete boundaries, which move as wholes along continuous paths and retain their identity upon colliding with each other; individuated objects high on the animacy scale (*boy, cat*).

- **MASS : COUNT ≠ STUFF : OBJECT**
  There are also many mismatches between the linguistic mass/count distinction, the conceptual 'stuff'/'object' distinction, and the structure of the matter in the world:
  - INTRA-LINGUISTIC VARIATION in the mass/count encoding
    - In a single language, different types of entities that come in natural units of equal perceptual salience may differ as to whether they are encoded as mass or count:

      rice [−C]  *lentil/lentils* [+C] 

    - *foliage/leaves, footware/shoes, change/coins, drapery/curtains, mail/letters*
      (lexical mass/count doublets, near synonyms, see Quine 1960, p.91; McCawley 1975)
      Example: *rake leaves into a pile* versus **rake foliage into a pile* (Grimm 2012)
    - *carpeting/carpets, drapery/drapes (*two lexical items based on the same root are related by morphological operation; one has a mass use and the other has a count use)
    - *hair/hairs, rope/ropes, stone/stones* (a single lexical item can be realized as mass or count)
      Example: *A man with hair on his head has more hair than a man with hairs on his head.* (Richard Lederer, *Crazy English*)
  - CROSS-LINGUISTIC VARIATION in the mass/count encoding
    Mass expressions in one language have count near-synonyms in another:

    furniture ENGLISH — *le meuble / les meubles* FRENCH
    hrách CZECH, *ropóx* RUSSIAN — *pea / peas* ENGLISH, *pisello / piselli* ITALIAN

- **MASS : COUNT ≠ STUFF : OBJECT**
  independence of the linguistic and conceptual (pre-linguistic) distinctions

  Barner and Snedeker (2005): *Who has more X?* quantity judgements

count Ns mass Ns

"object" mass Ns prototypical mass Ns



Who has more shoes? *Who has more furniture?* *Who has more mud?*

number-based judgements volume/mass-based judgements

- The mass/count distincton is asymmetric (form-meaning mappings):
  - count syntax specifies quantification over individuals
  - mass syntax is unspecified: it does NOT force a construal of objects as unindividuated: "object" mass nouns (*furniture*) can be used to refer to individuated things, they pattern with count nouns in so far as they allow for number-based comparison judgements.
- Argument against Quine's (1960) view (adopted in Link 1983, 1998; Bloom 1994,1999 i.a.) that **only** count nouns denote discrete individuals, that is, *chairs* divides reference, but *furniture* does not.

1.4 Main questions (1st part of the course)

i. What grounds the mass/count distinction?

Is the mass/count distinction linguistic (stemming from grammar) or rooted in language independent cognitive systems?

How do the grammatical manifestations of the mass/count distinction relate to the conceptual, pre-linguistic contrast 'undifferentiated stuff versus individuated object'?

How do the grammatical manifestations of the mass/count distinction relate to things in the real world?

ii. How do we motivate the variation in the mass/count lexicalization patterns?

2 Mereology

2.1 Link (1983): Atomic and non-atomic semilattices for mass/count/plural

2 Mereology

- Natural language semantics
  - long-standing tradition of applying mereology to the semantics of nominal domain:
    - Foundations: Link 1983, 1987, 1998; Krifka 1986, 1989, 1995,1998;
    - Further developments: Landman 1991, 1996, 2000, 2011; Chierchia 1998, 2010; Rothstein 2010; Champollion 2010, 2015; Pelletier 2012; Sutton and Filip 2016a,b,c,d,e (among many others);
  - extension of the mereological tools to the verbal domain:
    - Foundations: Bach 1981, 1986; Krifka 1986,1989.
    - Contemporary mereological, and also scalar approaches, to aspect phenomena.

- Historical background
  - Mereology, a theory of part-whole relations, from the Greek *μέρος* (meros), meaning "part" (also "portion", "segment")
  - Origins: the Pre-Socratics (6th and 5th century BC, see Varzi 2011)
  - Mereology started as an alternative to set theory (originally proposed by Leśniewski 1916; reinvented by Leonard and Goodman 1940; Goodman 1951).
    - Set theory: subset relation $\subseteq$ and element-of relation $\in$.
    - Mereology: the basic relation of part-of $\leq$.

Surveys: Simons 1987; Casati and Varzi 1999; Varzi 2010.

2.1 Link (1983): Atomic and non-atomic semilattices for mass/count/plural

- **Goal**

An analysis of mass terms and plurals, capturing both the
– similarities between MASS and PLURAL and
– differences between MASS and COUNT,
based on certain intuitions about the part-whole relations of their denotations. *

- **Main innovation**

Giving more structure to the domain of individuals. In addition to

– ordinary individuals like John, Mary, this chair, Mary's ring (as in standard interpretations of the predicate calculus or in Montague Grammar), also
– plural individuals like *John and Mary* together or *the children* which have the ontological status of ordinary individuals (type e);
– quantities of stuff or matter that correspond to the stuff that makes up John, the plural individual John and Mary together or the gold that makes up Mary's ring.

* Massey (1976): another theory of plurals and mass nouns formulated in a mereological framework. Early approaches to plural semantics adopted set theory (Hausser, 1974; Bennett, 1974).

- **Empirical motivation**

Collective and distributive interpretations: entailment relations between wholes/collections and their proper subparts

DISTRIBUTIVITY: The property *P* distributes from wholes—plurality or stuff—to their proper parts

(1) a. The apples in this basket are red. distributive predicate *red*
⇒ The apple taken from this basket is red.
b. The coffee in the cup is warm. distributive predicate *warm*
⇒The coffee at the bottom of the cup is warm.

(2) John and Mary yawned. ⇒ John yawned and Mary yawned. distributive predicate *yawn*

COLLECTIVITY: The property *P* can only be attributed to wholes—plurality or stuff

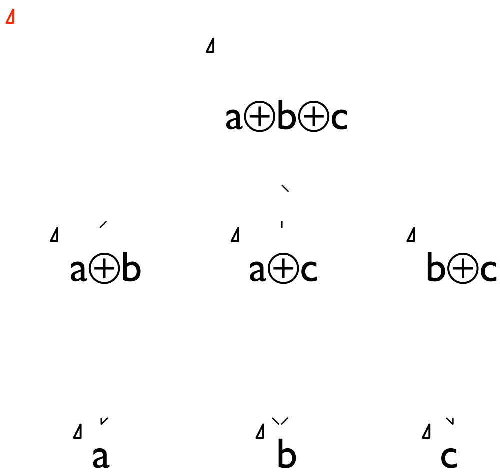
(3) a. *The **child** gathered around the pool. collective predicate *gather*
b. The crowd gathered around the pool.
c. The water gathered in the pool.
d. The children gathered around the pool.

DISTRIBUTIVE/COLLECTIVE ambiguity with plural terms:

(4) The women lifted the piano. "mixed" predicate *lift*
(i) Each woman individually ...
(ii) All the women together ...

- The domain of entities is not an unstructured set, but has the algebraic structure of a JOIN SEMILATTICE that models the mereological part-whole relations. The 'join' corresponds to the mereological sum operation '⊕', taken as basic; the part relation '≤' is derived from it.

- We have two ontological domains, *I* for individual objects and *M* for quantities of matter.



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- The domain *I* of individuals contains individuals of ordinary sort like John, Mary and the chair at my desk. Suppose that these are atoms of *I* = {*a, b, c*}. We extend *I* by means of the binary sum operation ⊕ to its superset *S* of sum (plural) individuals: $I \subseteq S$.

a⊕b⊕c

a⊕b a⊕c b⊕c

a b c

⊕	binary **sum** operation
a⊕b	sum (sum individual consisting of *a* and *b*)
≤	**part** relation: $\forall x,y \in S\ [x \leq y \leftrightarrow x \oplus y = y]$
a ≤ a⊕b	*a* is a part of the sum consisting of *a* and *b*
a ≤ a	*a* is part of itself

< **proper part** relation: $\forall x,y \in S\ [x < y \leftrightarrow x \leq y \wedge x \neq y]$

a < a⊕b *a* is a proper part of the sum of *a* and *b*

⊗ **overlap** relation: $\forall x,y,z \in S\ [x \otimes y \leftrightarrow \exists z \in S[z \leq x \wedge z \leq y]]$

a⊕b ⊗ a⊕c

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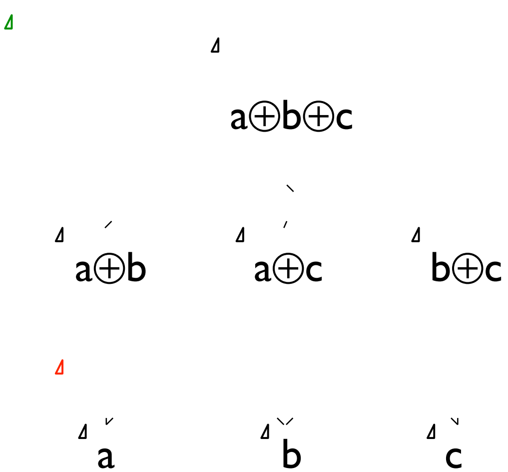

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$a \oplus b \oplus c$

$a \oplus b$ $a \oplus c$ $b \oplus c$

a b c

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≤ $\quad$ ≤

a $\quad$ b $\quad$ c

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<

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$a\oplus b \otimes a\oplus c$ overlap in a

Atomic join semilattice for singular and plural count nouns

- ATOMS are elements at the bottom of an atomic semilattice; the smallest, minimal parts defined as elements
  - that have no proper parts: $\text{ATOM}(x) =_{def} \neg\exists y(y < x)$
  - are used to represent the denotation of singular forms: definite singular expressions (*John* or *this chair*) and indefinite singular expressions (*a chair, one chair*).

- Non-atomic elements--the SUM INDIVIDUALS--model the meanings of plural forms: e.g., *chairs, two chairs, the chairs.*
  - Algebraic (upward) closure under sum operation ⊕ for the meaning of PLURAL forms:

    $*P =_{def} \{x \mid \exists P' \; P' \neq \emptyset \wedge P' \subseteq P \wedge x = \oplus P']\}$ '*' the star operator

    The algebraic (upward) closure $*P$ of a set P is the set that contains everything that is

    (i) a P (denotation of a singular form) or

    (ii) a sum consisting of two or more Ps

    = isomophic to the powerset of P minus the empty set.

    Our example: $[\![*P]\!] = \{a, b, c, a\oplus b, b\oplus c, a\oplus c, a\oplus b\oplus c\}$

$a\oplus b\oplus c$

$a\oplus b$ $a\oplus c$ $b\oplus c$

a b c

  - exclusive view of $[\![N\mathit{pl}]\!]$:
    the plural form means *two or more Ns,* atoms excluded from plural denotations
    $[\![N\mathit{pl}]\!] = \{a\oplus b, b\oplus c, a\oplus c, a\oplus b\oplus c\}$
    $[\![Nsg]\!] = \{a, b, c\}$

Champollion and Krifka 2016, p.523, 526

Post-Link (1983) developments: Three different views on the denotation of plurals

$a \oplus b \oplus c$

$a \oplus b$ $a \oplus c$ $b \oplus c$ Plural exclusive

a b c Singular

Plural inclusive

Exclusive: the plural form means *two or more Ns* (Link 1983, Chierchia 1998)
singular and plural forms of a count noun denote disjoint sets
$[\![N\mathit{sg}]\!] = \{a, b, c\}$ $[\![N\mathit{pl}]\!] = \{a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c\}$

Inclusive: the plural form means *one or more N*, denotes its algebraic closure, is number neutral (Krifka 1986, Sauerland 2003, Sauerland et al 2005, Chierchia 2010)
$[\![N\mathit{sg}]\!] = \{a, b, c\}$ $[\![N\mathit{pl}]\!] = {*}[\![N\mathit{sg}]\!] = \{a, b, c, a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c\}$

Mixed: the plural form is ambiguous between *one or more N* and *two or more Ns* (Farkas and de Swart 2010)

- Argument in support of the inclusive view: we cannot exclude atoms from plurals

(1) There are no cats in the room.

Evidence: the semantics of plural negative quantified Determiner Phrases (DPs) like *no cats* (Schwarzschild, 1996, p. 5; Chierchia 2010).
Intuitively, (1) is false if there is exactly one cat in the room. However, the theory predicts that (1) is true if there is exactly one cat, i.e., no group of cats (empty set), contrary to our intuitions. The proper analysis of sentences like (1) requires that plurals include atoms.

(2) A: Do you have children?
B: Yes, one. / *No, (just) one.

(3) A: Do you have more than one child?
B: *Yes, one. / No, (just) one.

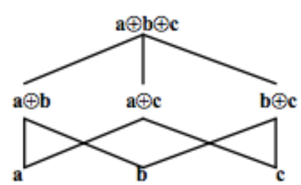
- Problem: Why cannot positive sentences like *there are cats on the mat* be used to describe a situation in which there is only one cat on the mat?
  - Krifka (1986) argues that the singular form blocks the plural form via competition: On the inclusive view of plurals, when singular reference is intended, singular and plural forms are in pragmatic competition, and the more specific singular form blocks the plural form.
  - Chierchia (2010) suggests that that this effect might be due to a scalar implicature.

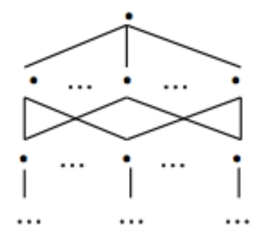
Ontological distinction between individuals and matter - two disjoint domains:

- Atomic domain of individuals I_i.
- Non-atomic domain of quantities of stuff or matter M_i (lack of the atomicity requirement).
- Materialization function h: $I_i \rightarrow M_i$ maps individuals (atomic and plural) to the quantities of matter they consist of, whereby h is a homomorphism: $h(x \oplus_i y) = h(x) \oplus_m h(y)$.

Every count predicate ***P*** has a mass term correspondent ^{m}***P*** which denotes a set of quantities of matter in M_i.

⟦*chair*⟧ / ⟦*chairs*⟧ ⟦*wood*⟧





i-sum ("individual sum"): $\oplus_i$ m-sum ("material sum"): $\oplus_m$

i-part ("individual part"): $\leq_i$ m-part ("material part"): $\leq_m$

- Count nouns (*chair*) and "object" mass nouns (*furniture*) denote individuals, they take their denotation from an atomic join semilattice: e.g. ⟦*chair*⟧ $\subseteq I_i$, ⟦*furniture*⟧ $\subseteq I_i$ (Link 1991/1998, p.214)
- Mass nouns denote quantities of matter, they take their denotation from a non-atomic join semilattice: e.g. ⟦*wood*⟧ $\subseteq M_i$

- Motivation for the materialization function *h*: Individuals and the stuff they consist of may have distinct properties.

  This ring is ***new****, but the gold (that this ring consists of) is* ***not new****.*

  Interpretation:
  The *x* such that *x* makes up this ring and is gold and is old but this ring is not old.

  Not a contradiction:
  – *x* and this ring are not the same thing, *x* is the value of *h* with *this ring* as its argument;
  – *h* allows for two things occupying the same place at the same time to overlap materially, but not individually;
  – no violation of Leibniz's *Identity of Indiscernibles*: $\forall x \forall y\ [x = y \leftrightarrow \forall P(Px \leftrightarrow Py)]$ 'two or more entities are identical, if they have all their properties in common'

- "Our guide in ontological matters has to be language itself ... individuals are created by linguistic expressions involving different structures even if the portion of matter making them up is the same" (Link 1983, p. 303-4).

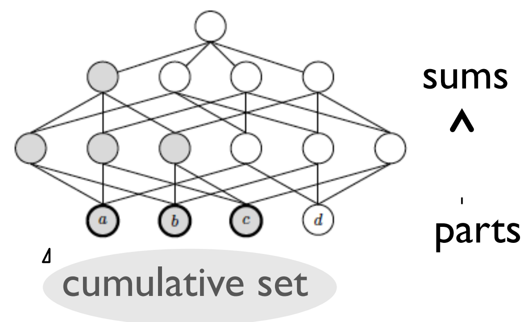
- Second-order mereological properties of predicates
  - cumulativity
  - divisivity

CUMULATIVITY : signature property shared by plurals and mass terms

CUMULATIVE(P) $=_{def}$ $\forall x,y[P(x) \wedge P(y) \rightarrow P(x \oplus y)]$

A predicate *P* is *cumulative* iff, whenever *P* applies to any *x* and *y*, it also applies to their sum.
(The "upward" closure property.)

Champollion and Krifka 2016, p.524



Presupposition: CARD(P) > 2. $\exists x,y[P(x) \wedge P(y) \wedge \neg\, x = y]]$.
For any *x* and *y* to which *P* applies, assume that they are two distinct entities, otherwise, predicates would coincide.

The property of "cumulative reference" (the term introduced by Quine 1960, p. 91)

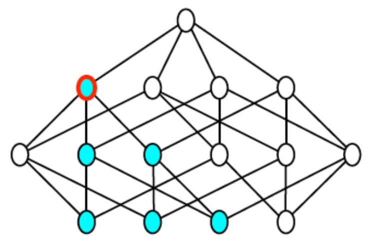
- holds true for
  - mass nouns like *water* ("any sum of parts which are water is water", see Quine 1960, p. 91, following Goodman 1951) and "object" mass nouns like *furniture* (Quine (1960);
  - bare plurals like *horses*: "If the animals in this camp are horses, and the animals in that camp are horses, then the animals in both camps are horses" (Link 1983, p.303);
    Plural formation results in cumulative atomic predicates: CUM(*⟦*horse*⟧).
- does not hold for singular count nouns (*an apple*): a sum of two or more apples is not *an apple*.

CLOSURE of predicate denotations under sum formation and maximal individual

- A unified interpretation for expressions that are insensitive to atomicity, but sensitive to formal properties (like having a supremum) that are common to both the COUNT (singular and plural) and the MASS algebra. Example: determiners like *the, all, some* and *no*.

- A case in point: the definite article *the*

  the water
  the chairs supremum ($\llbracket P \rrbracket$)
  the chair



  supremum ($\llbracket P \rrbracket$): "the greatest or maximal individual in $\llbracket P \rrbracket$" in the relevant situation, where $\llbracket P \rrbracket$ is a (sub)set of entities in the denotation of *P* (Sharvy 1980, Link 1983, following Montague 1973).

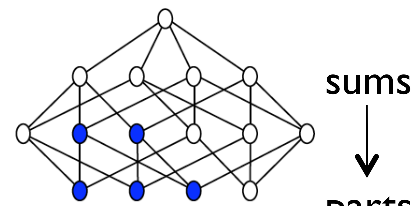
- Advantage: a uniform treatment of the definite article that
  (i) explains why it is defined for both count (singular and plural) and mass nouns,
  (ii) *the NP(s)* is treated as entity-denoting rather than explicitly quantificational,
  (iii) the Frege-Russell theory of definite descriptions subsumed as a special case: on Link's (and Sharvy's) account, the iota-terms for singular definites ($\exists y\ [y=\iota x Px \wedge Qy]$) do not require the stipulation of the "one and only one" condition, the iota-terms are treated as a special case of the supremum σ-terms ($\exists y\ [y=\sigma^{*}xPx \wedge Qy]$; where $\sigma^{*}xPx$ carries the presupposition that there are at least two P's) (Link 1983, p. 307) which denote uniquely specified i-sums, which are the suprema of the extensions of the predicates in question.

DIVISIVITY and the minimal parts problem

DIVISIVE(P) $=_{def}$ $\forall x[P(x) \rightarrow \forall y[y < x \rightarrow P(y)]]$

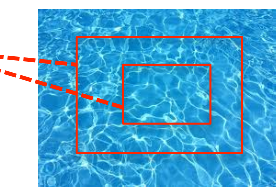
Grimm 2012, p.112; Champollion & Krifka 2016, p. 525

A predicate *P* is divisive if and only if whenever it holds of something, it also holds of each of its proper parts. (The "downward" closure property').

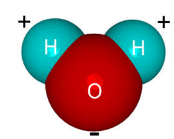


Cheng's condition (Cheng 1973)
Precursors: Aristotle *Metaphysics* 1016b17-24; 1052a32), Frege 1884, p.66

- The divisivity property
  - does not hold for singular count nouns (*apple, boy*), plurals (*apples*) and for aggregate/ collective mass nouns (*furniture*).
  - is often assumed to hold for the mass noun denotations like ⟦*water*⟧ "A part of water is still water."



- The MINIMAL PARTS problem:
  - There are parts of water too small to count as water, namely hydrogen and oxygen atoms (Quine 1960, p.98);
  - heterogeneous stuff like fruitcake has proper parts like individual sultanas not describable as *fruitcake* (Taylor 1977).



- Link (1983) avoids any reliance on divisivity, because of the unresolved minimal parts problem; mass terms take their denotations from the non-atomic domain, i.e., it may but need not have atoms (see Partee 1999, p.95); agnostic wrt the minimal parts problem.

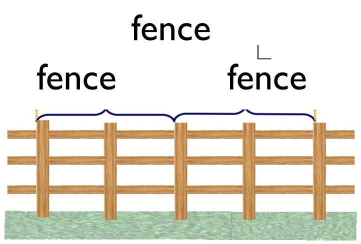
Strictly speaking, no *P* is divisive in its reference (Gillon 1992, i.a.)
I.e., no *P* is arbitrarily and infinitely divisible into parts, each of which is still describable by *P*. No principled reason to maintain that mass nouns are divisive, but count nouns are not.

Weakened divisivity: A divisive predicate *P* is true of certain minimal parts of a thing of which *P* is true.

❖ Problem 1: What is the minimal part in the denotation of *water, salt, meat, fruitcake*?

- » "generally an empirical matter" (Pelletier 1975, p.453),
- » contextually determined and coarser than the part structures a scientist would ascribe to some matter (Moltmann 1991),
- » set by a "granularity parameter" in the definition of divisivity (Champollion 2010).

❖ Problem 2: Count nouns like *fence, twig, rope, wall* are weakly divisible (Zucchi and White 1996, 2001): A fence divides into fences, line divides into lines. Only "sortal" count nouns like *cat* are never divisive in their reference, under some idealization. A weakened divisivity property is NOT SUFFICIENT to distinguish mass from count denotations.



Context-sensitive proposals: Count nouns like *fence* have meanings partially specified by context, and denote indivisible predicates in a given context (Chierchia 2010, Rothstein 2010).

3 From Link's DOUBLE DOMAIN approach to ONE DOMAIN approaches

2) ATOMICITY neither sufficient nor necessary for the [+C] status of a noun
   - mass nouns like *furniture* have inherently individuable, indivisible objects—natural atoms—in their denotation (e.g., individual chairs);
   - count nouns like *fence, wall, ribbon* do not have inherently individuable objects in their denotation.
3) NON-ATOMIC domain insufficient to model the properties of "object" mass nouns like *furniture*:
   - the denotation of *furniture* is no less "atomic" than the denotation of *a piece of furniture* or *chair,* so *furniture* cannot be treated on a par with prototypical mass nouns like *water* ((Chierchia 1998a, p.68) and *table* or *chair* Chierchia 2010, p.140);
   - grammatically, *furniture*-like and *water*-like mass nouns exhibit different properties (e.g., *big furniture, big apples* vs ? *big water, see* Stubborn Distributivity, Schwarzschild 2011);
   - unclear how to obviate the infinite regress in the application of the divisivity property (e.g., water cannot be infinitely/arbitrarily subdivided into subparts while preserving their quality as water) ('homeopathic' semantics in Landman's (2011) terms, see his criticism);
4) ATOMIC vs NON-ATOMIC disjoint ontology leads to counterintuitive predictions
   - Intra-linguistic mass/count variation: the mass/count lexical doublets like *footware/shoes, foliage/leaves, change/coins, drapery/curtains:*

     (i) mass nouns do not denote the material counterpart of the count noun denotations;

     (ii) counterintuitive to think that *footware* and *shoes,* for ex., differ in their ontological nature.
   - Cross-linguistic mass/count variation: *furniture* is mass in English and *le meuble/les meubles* count in French. "It is counterintuitive to think that your furniture changes its ontological nature by referring to it in English versus French" (Chierchia 2010, p.144).

3 From Link's Double Domain[1] approach to One Domain approaches

Some arguments against the two-sorted domain for entities

1) The snowman puzzle (Bach 1986):

*The snow making up this snowman is quite **new** but the H_2O making it up is very **old** (and the H and O even older!)*

Interpretation: The *x* such that *x* constitutes the snowman and *x* is snow is quite NEW but the *y* such that *y* constitutes the snowman and *y* is water is very OLD (not new).

! Contradiction: *x* (snow) and *y* (water) must be identical, because *h* is a function, and yet *x* and *y* have contradictory properties: x (snow) is new and y (water) is old. Two things with contradictory properties cannot be identical (see Leibniz's *Principle of Identity of Indiscernibles*).

Bach's (1986, p.14) suggestion: "remove altogether the entities in *D* [the non-atomic domain containing the portions of matter or stuff] from the domain of individuals" (see later atomistic mereologies)

[1] Chierchia's (2010) term

One Domain Approaches

- **Atomistic mereologies**: an additional axiom requiring everything in the domain be composed of atoms (see below).
  - Landman (1989, 2011): sets used to model sum individuals; ordinary set theory implies a commitment to everything ultimately being composed of atoms, mereological atoms are represented by singleton sets (see also Scha 1981, Schwarzschild 1996).
  - Chierchia (1998, 2010)
  - Rothstein (2010, 2016)

- **Mereologies that are undetermined with respect to atomicity**
  No additional axiom of atomicity ('All things are made up of atoms') or axiom of atomlessness ('Everything is infinitely divisible') are postulated.
  Advantage: "we do not have to commit ourselves to an atomic or a non-atomic conception of the world" (Krifka 1989, p.81).
  - Krifka 1986-present
  - Sutton and Filip (2016a,b,c,d, e)

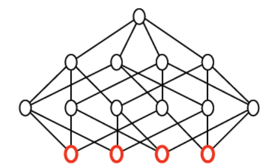
Atomistic mereologies (a short preview)

Atomistic mereologies

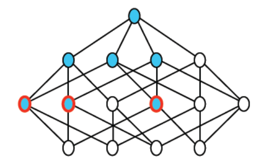
ATOMICITY axiom: $\forall x \exists y[y \leq x \wedge \neg \exists z(z < y)]$ or $\forall y \exists x[\text{atom}(x) \wedge x \leq y]$
All things are made up of atoms.

"since in subdividing something we always get to an end, there is no principled reason to maintain that mass nouns (even those whose granularity is unclear) do not have an atomic structure" (Chierchia 1998a, p.68).

FORMAL ATOM: $\text{ATOM}(x) =_{def} \neg \exists y(y < x)$ Champollion and Krifka 2016, p.523
An atom is an individual which has no proper parts. (Atom relative to a domain.)



P-ATOMICITY: Atoms defined relative to a property *P* (Krifka 2007):
$\text{ATOM}(x,P) = P(x) \wedge \neg \exists y[y < x \wedge P(y)]$
An atom relative to a property *P* applies to *x*, not to any proper part of *x*.



NATURAL ATOM: Intuitively, inherently individuable entity; neither necessary (*fence*) nor sufficient (*furniture*) for being a count noun.

For each mass noun there are minimal elements (formal atoms in a Boolean algebra) in its denotation, just like for count nouns: see Chierchia 1998a,b; Rothstein 2010; Landman 2011.

(1) If ***all*** nouns are interpreted with respect to an atomic Boolean algebra, why can only the denotation of count nouns be directly counted? Why cannot we directly count the denotation of mass nouns? Why is # *three muds* odd or highly marked?

(2) What is it about the nature of atoms (minimal elements) in the denotation of mass nouns that prevents their direct counting?

‘One domain’ mereologies that are undetermined with respect to atomicity

Origins: Krifka (1986,1989 and elsewhere)

One mereologically structured domain and measure functions: Krifka (1986-present)

- Quine (1960): the notion of a count noun involves "built in modes of dividing their reference"

  "To learn 'apple' it is not sufficient to learn how much of what goes on counts as apple; we must learn how much counts as *an* apple, and how much as another (...). Such terms possess built-in modes (...) of dividing their reference" (Quine 1960, p.91).

  Implicit idea: Learning the meaning of *an apple* involves the knowledge about how we **INDIVIDUATE** one 'apple-sized unit of stuff', by some inherent unit of measurement.

- Quine's notion of "built-in modes of dividing reference"
  - cannot be atomicity of predicates - Krifka's criticism of Link (1983).

    Reason: being an atomic predicate is insufficient for being a count noun, as object mass nouns like *footware* denote atomic predicates, just like count Ns like *shoes* (e.g., Krifka 2007).

  - is to be represented by means of **MEASURE FUNCTIONS** that introduce a **quantitative "criterion of applicability/application"** of predicates ("what is ONE in their denotation").

Proposal: Various **extensive measure functions** are defined on a single domain of objects structured by a complete join semi-lattice which is undetermined wrt atomicity.

Measure terms, count nouns and mass nouns

- Measure terms only have a quantitative criterion of application:
  $[\![\textit{pound}]\!] = \lambda n \lambda x[\textsc{pound}(x) = n]$
  - require a cumulative predicate and derive a quantized predicate:
    $[\![\textit{pound}]\!] = \lambda P.\textsc{cumulative}(P)\ \lambda n \lambda x[P(x) \wedge \textsc{pound}(x) = n]$
    "cumulative presupposition"
- Count nouns have a qualitative and a quantitative criterion of application:
  $[\![\textit{apple}]\!] = \lambda n \lambda x[x \text{ is apple} \wedge \textsc{natural unit}([\![\textit{apple}]\!])(x) = n]$
  - The quantitative criterion is represented by the NATURAL UNIT measure function (also # used, see Krifka 2008) which determines the atomic or singular non-overlapping objects in their denotation.
- Mass noun denotations only have a qualitative criterion of application:
  $[\![\textit{water}]\!] = \lambda x[x \text{ is water}]$

  (but see Sutton & Filip 2016 b, c, d, e for a view on which mass is due to the individuation criterion (similar to Krifka's quantitative criterion) not being satisfied for mass denotations)

Summary and formulas based on Krifka 2007

Extensive measure function

- expressed by measure terms: *pound, meter, kilogram, liter, ...*
- μ is an EXTENSIVE MEASURE FUNCTION on a part structure *P*, and relative to the sum operation ⊕, iff it maps substances to positive real numbers such that: $\neg x \otimes y \rightarrow [\mu(x \oplus y) = \mu(x) + \mu(y)]$ (additivity) (Krifka 1989).

In physics and measurement theory, an extensive measure function is one whose magnitude is additive (see Krantz et al., 1971; Cohen et al., 2007):
POUND measure function which measures weight is additive:

- 3 pounds of apples + 5 pounds of apples = 8 pounds of apples;
- the measure function *pound* is extensive on the part structure of apples, because it tracks its part structure: proper parts of some quantity of apples have a lower weight, and its superparts higher weight.

- DEGREE CELSIUS measure function which measures temperature is not additive:
  - If a quantity of water has 60°C and another quantity 20°C, they do not add up to a quantity of water that has 80°C.
  - If a quantity of water has a certain temperature, there is no reason to expect that proper parts of it will have lower temperatures and its superparts higher temperatures.

Measure functions like DEGREE CELSIUS are called intensive measure functions.

NOMINAL MEASURE PHRASE
EXTENSIVE measure function

a. two pounds of apples
b. *forty degrees Celsius of water

COMPOUND
INTENSIVE measure function Krifka 1989, 1998

c. *two pound(s) apples
d. ten degree Celsius water

- Nominal measure phrases (or pseudopartitives) are formed with terms for extensive measure functions like *pound*.
- Compounds are formed with terms for intensive measure functions like *degree celsius*.

Krifka 1989; Schwarzschild 2002, 2006; Braseovanu 2009, i.a.
Schwarzschild (2002, 2006) uses a monotonic measurement (or dimension): μ is a monotonic measure function with respect to a part structure iff: for individuals *x, y*, if *x* is a proper part of *y*, then the measure of *x* is smaller than that of *y*, $\mu(x) < \mu(y)$.

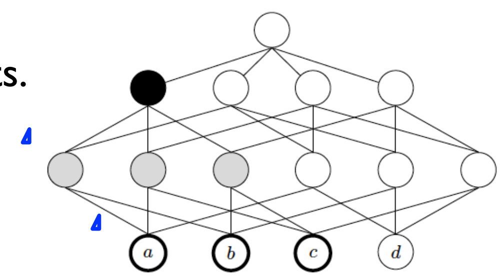
Cumulative and quantized predicates, and measure functions

- CUMULATIVE(P) $=_{def}$ $\forall x,y[P(x) \wedge P(y) \rightarrow P(x \oplus y)]$ "cumulative" Quine 1960
  Cumulative nominal predicates are expressed by
  (i) lexical **mass nouns** (*water*),
  (ii) bare plurals (*apples)*

Quantized predicates are derived from cumulative predicates by means of extensive **measure functions**, expressed by measure terms.

- QUANTIZED(P) $=_{def}$ $\forall x,y[P(x) \wedge y < x \rightarrow \neg P(y)]$ "quantized" Krifka 1986, 1989
  A predicate P is quantized if and only if whenever
  it holds of something, it does not hold of any of its proper parts.

  Quantized nominal predicates are expressed by
  (i) lexical **count nouns** (*an apple*),
  (ii) numerical NPs (*three apples*),
  (iii) measure NPs (*three pounds of apples, three liters of water)*



quantized sets

No proper part of an entity that falls under *an apple* should fall under *an apple*.
No proper part of an entity that falls under *three liters of milk* also falls under *three liters of milk*.

Measure construction (Krifka 1989, 1995, 2004)

SEMANTICS: $[\![\textit{three liters of water}]\!]$
$= [\![\textit{three liters}]\!]([\![\textit{water}]\!])$
$= \lambda P \lambda x[P(x) \wedge \text{LITER}(x) = 3](\lambda x[\text{WATER}(x)])$
$= \lambda x[\text{WATER}(x) \wedge \text{LITER}(x) = 3]$
A quantized predicate referring to sums of water to the amount of three liters.

- The measure phrase *three liters* functions as an operator on the mass noun *gold*, which is cumulative, and yields a quantized predicate: No proper part of an entity that falls under *three liters of water* also falls under *three liters of water*.

- A measure phrase is formed by means of an extensive measure function expressed by a measure word (*liter*) that is applied to a number expressed by a numeral (*three*); number words are arguments for measure functions.

SYNTAX [three liters] (of) water$_{N}$

three liters$_{N/N}$ water$_{N}$

liters$_{N/N,NM}$ three$_{NM}$

Count noun construction (Krifka 1989, 1995, 2004)

$[\![apple]\!] = \lambda n \lambda x[[\![apple]\!](n)(x)]$
$= \lambda n \lambda x[x \text{ is apple} \wedge \text{NATURAL UNIT}([\![apple]\!])(x) = n]$ Krifka 2007

- Lexical count semantic predicates are subsumed under quantized predicates.
- Count nouns in English like *(an) apple* are inherently relational: two-place relations between numbers and entities, ignoring their internal constituency (Krifka 1995, p.406).
- Count nouns in English contain a built-in extensive measure function NATURAL UNIT (Krifka 1989, 1995):
  - an additive measure function that is related to the meaning of *P* mapping a property to the natural unit of this property; or
  - maps an individual *x* that counts as one *P* to ***1*** (Krifka 2007)

$[\![apples]\!] = \lambda x \exists n[[\![apple]\!](n)(x)]$
$= \lambda x \exists n[x \text{ is apple} \wedge \text{NATURAL UNIT}([\![apple]\!])(x) = n]$

- Bare plural NPs - semantic plural: the number argument *n* of a count noun is bound by an existential quantifier (*apples* as in *Mary ate apples yesterday*);
- The selection of singular/plural forms is by syntactic agreement and without semantic import. Evidence: general plural agreement with decimal fractions (a) and lack of number agreement in Turkish (b): a. one point zero inhabitants / *inhabitant (per square kilometer).
  b. üc kiz / *kizlar 'three girl / girls'

Counting constructions with numerical words like *two* (Krifka 1989, 1995, 2004)

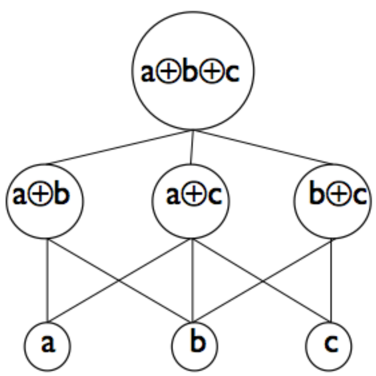
Numerical words can only combine with a quantized predicate and derive a quantized predicate.

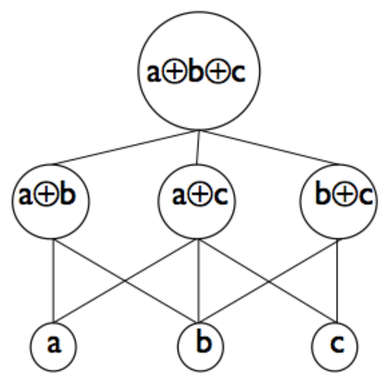
- Count nouns like *(an) apple* are inherently quantized in their lexical meaning, because their lexical representation contains a measure function NATURAL UNIT. (Numbers as arguments!)

⟦*two apples*⟧ = ⟦*apples*⟧(⟦*two*⟧)
= λnλx[x is apple ∧ NATURAL UNIT(⟦*apple*⟧)(x) = n](2)
= λx[x is apple ∧ NATURAL UNIT(⟦*apple*⟧)(x) = 2]
A quantized predicate referring to sum individuals of two apples.

⟦*apples*⟧ = {a, b, c, a⊕b, a⊕c, b⊕c, a⊕b⊕c}

⟦*two apples*⟧ = {a⊕b, a⊕c, b⊕c}





Counting constructions with numerical words like *two* (Krifka 1989, 1995, 2004)

- Why **two mail(-s)* or **two silverware(s)*?

  Object mass nouns like *mail* have the same structural meaning as *letter(s)*: they apply to a set of atomic objects and sums of atomic objects. This predicts that expressions like **two mail(-s)* should be fine, but they are not (Krifka 2008).

  The essential feature is quantization (akin to Quine's "built-in modes of dividing reference"), not atomicity: *letter* has a 'built-in' criterion of counting (a measure function NATURAL UNIT) that *mail* lacks, λx[x is mail], which also means that semantic plural is not applicable (Krifka 2007).

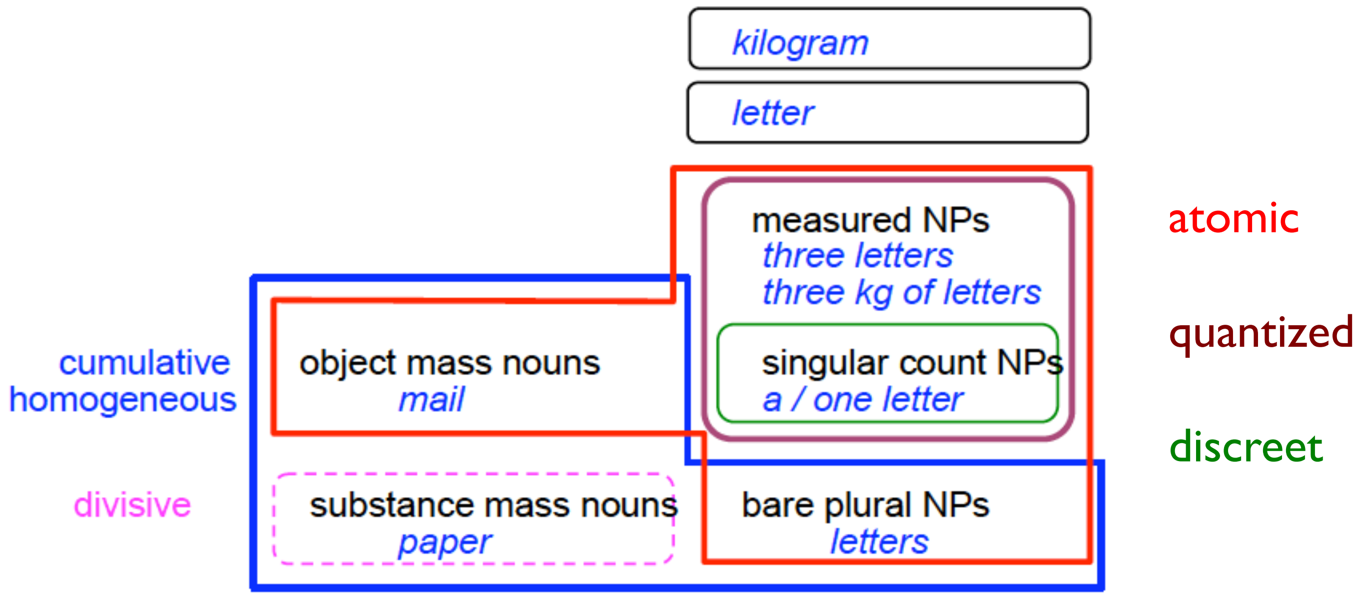
  Object mass nouns (*mail*) and substance mass nouns (*water*) require measure terms in counting constructions, as they are not inherently quantized:
  two pieces of mail, two liters of water;
  two waters (= two [SPECIFIED PORTIONS OF] water) (shift)

Summary: Krifka's theory of the mass/count distinction

- The essential feature that grounds the mass/count distinction is quantization, not atomicity.
- Count semantic predicates are represented by quantized predicates, which are built by means of extensive measure functions requiring cumulative predicates as arguments.
- Count nouns in English like *(an) apple* do not refer to atoms, but contain a built-in measure function, called NATURAL UNIT (Krifka 1986, see also Champollion and Krifka 2016, p.531):
  = $\lambda n \lambda x$[x is apple $\wedge$ NATURAL UNIT($[\![$*apple*$]\!]$)(x) = n]

  The internal constituency of count nouns in English is that of a classifier construction. The main difference to classifier constructions, as in Mandarin Chinese, is that the reference to a natural unit (the quantitative criterion of application) is built into the head noun, making it a count noun (Krifka 1989, p.84).
- Numerical words (*five*) require a quantized predicate, generate a quantized predicate.
- Object mass noun (*mail*) and substance mass noun (*water*) denotations cannot be combined with numerical words, because they are not quantized in their lexical meaning; they require measure terms to be counted. (Atomicity of *mail* is insufficient for counting.)
- The main difference between measure constructions and classifier constructions is that in the latter the measure function depends on the head noun.
  (1) measure construction: $[\![$*five pounds of gold*$]\!]$ = λx [gold(x) $\wedge$ pound(x) = 5]
  (2) classifier construction: $[\![$*five head of cattle*$]\!]$ = λx[cattle(x) $\wedge$ NATURAL UNIT(cattle)(x) = 5]
  (3) counting construction: $[\![$*five apples*$]\!]$ = λx[apple(x) $\wedge$ NATURAL UNIT(apple)(x) = 5]

Overview of Krifka's nominal types



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Appendix

- Algebraic semantics mostly models unstructured parthood Champollion and Krifka 2016

Whole	**Part ("an arbitrary slice")**
John and Mary	John
some horses	a subset of them
the water in our pool	the water at the bottom of our pool
some jumping events	a subset of them
a running event from A to B	its part from A halfway towards B
a temporal interval	its initial half
a spatial interval	its northern half

- unstructured parts need not be cognitively salient parts of a whole, but may slice up the whole in arbitrary ways.
- unstructured parts are transitive, and hence may overlap

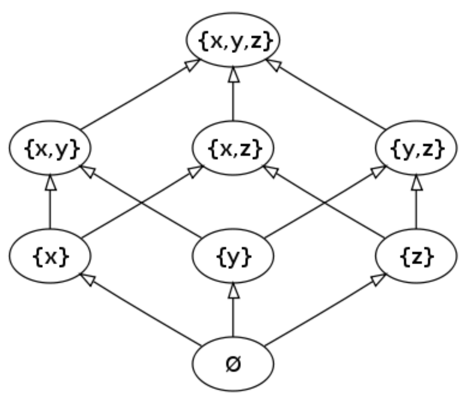
- Lexical semantics focuses on the analysis of structured parthood (e.g. Cruse 1986; see also overviews in Simons 1987; Varzi 2010)

Whole	**Part**	
a (certain) man	his head	Simons 1987
a (certain) tree	its trunk	
a house	its roof	
a mountain	its summit	
a battle	its opening shot	
an insect's life	its larval stage	
a novel	its first chapter	
a cognitively salient, integrated whole and not just a random collection of parts.	a cognitively salient part of a whole (meronymy, hyponymy) not transitive: **the thumb of this arm, *an arm without thumb*	

Boolean algebra

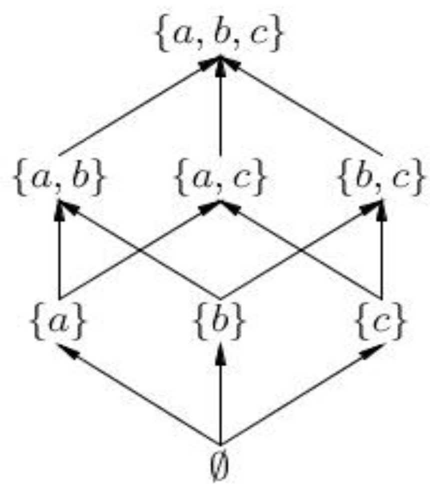
- uses variables like A, B, C, etc. The variables can take only two values TRUE and FALSE, or written as 1 and 0. In elementary algebra, the values of variables are numbers and the main operations are addition and multiplication.
- The main operations are
  - conjunction (or meet) $\wedge$,
  - disjunction $\vee$, and
  - negation $\neg$.

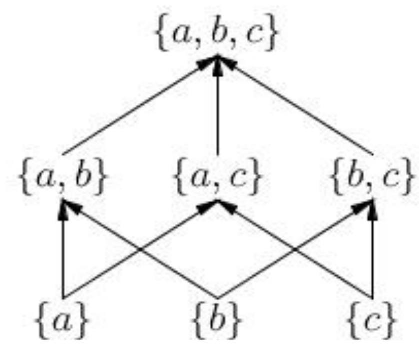
A common way of defining a Boolean algebra is as a lattice structure, a type of algebraic structure. Boolean lattice of subsets: e.g., $\{x,y\} \subset \{x, y, z\}$



No "null individual”

- The standard versions of CEM used in philosophy and semantic theory restrict the admissible algebraic structures to those that have no “null individual”, i.e., an individual which belongs to ALL other individuals in the way that the empty set is a member of all other sets in set theory.
- The existence of such a null individual is taken to be counterintuitive.
- Consequently, the structures that are assumed are a special type of lattice, a SEMILATTICE, an UPPER SEMILATTICE.





SEMILATTICE: Boolean algebra structure with the bottom null element removed

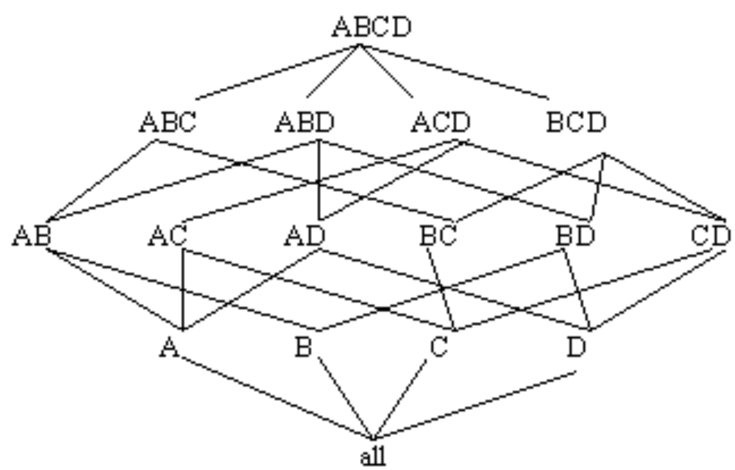
CLASSICAL EXTENSIONAL MEREOLOGY - SUMMARY

- Classical Extensional Mereology (CEM) consists of
  - THREE AXIOMS and requires only
  - a SINGLE PRIMITIVE NOTION in terms of which the rest of the mereological system can be defined.
- The three basic axioms are given in Lewis (1991) informally as follows:
  - AXIOM 1 (Unrestricted Composition): Whenever there are some objects, then there exists a mereological sum of those objects.
  - AXIOM 2 (Uniqueness of Composition): It never happens that the same objects have two different mereological sums.
  - AXIOM 3 (Transitivity): If *x* is part of some part of *y*, then *x* is part of *y*.
- The single primitive can be chosen to be
  - proper parthood $<$,
  - proper-or-improper parthood $\leq$,
  - sum $\oplus$,
  - overlap $\otimes$
  - disjointness.

  Other notions are definable in terms of whichever one is taken as primitive.

- AXIOM 1 (**Unrestricted Composition**): Whenever there are some objects, then there exists a mereological sum of those objects.
- Suppose the entire universe consists of
  - Ann (*A*),
  - Bill (*B*),
  - one car (*C*) and
  - one dog (*D*).

Then we may need represent all of their combinations, among which is Ann together with Bill (*AB*), which corresponds to the meaning of the conjunction *Ann and Bill*:



- AXIOM 2 (**Uniqueness of Composition**) excludes (1) and (2), because not every two elements have a unique sum.

(1) *a* *d*

b *c* *e* *f*

(2) *a* *c*

b *d*

- AXIOM 3 (**Transitivity**): {*a*} is a part of {*a*,*b*,*c*}, because it is a part of one its parts {*a*,*b*}.

(3) {*a*,*b*,*c*}

{*a*,*b*} {*a*,*c*} {*b*,*c*}

{*a*} {*b*} {*c*}

The part relation as the primitive notion and the sum operation defined from it.

- The **"part-of"** relation is reflexive, transitive and antisymmetric:

Axiom of reflexivity: $\forall x[x \leq x]$
Everything is part of itself.

Axiom of transitivity: $\forall x \forall y \forall z[x \leq y \wedge y \leq z \rightarrow x \leq z]$
Any part of any part of a thing is itself part of that thing.

Axiom of antisymmetry: $\forall x \forall y[x \leq y \wedge y \leq x \rightarrow x = y]$
Two distinct things cannot both be part of each other.

- Tarski (1929, 1956)
- Taking reflexivity (and antisymmetry) as constitutive of the meaning of 'part' simply amounts to regarding identity as a limit (improper) case of parthood.

The part '≤' relation is reflexive, which is at variance with how English *part* is used.
It is distinguished from the proper part relation, which is irreflexive '<'.
The relation of proper-part can be defined based on the part relation '≤':

- **proper-part-of relation <**
  The proper-part-of relation restricts parthood to nonequal pairs:
  $x < y =_{def} x \leq y \wedge x \neq y$
  A proper part of a thing is a part of it that is distinct from it.

  or

  $x < y =_{def} x \leq y \wedge \neg(y \leq x)$
  x is a proper part of a thing if it is a part of a thing which itself is not part of x.

The relation of overlap can be also defined based on the part relation '≤' :

- **overlap relation $\otimes$**
  $x \otimes y =_{def} \exists z[z \leq x \wedge z \leq y]$
  Two things overlap if and only if they have a part in common.

The sum operation ⊕

- The classical definition is due to Tarski (1929, 1956). (For other definitions, see Sharvy 1979, 1980, for instance.)

  $\text{sum}(x,P) =_{def} \forall y[P(y) \rightarrow y \leq x] \wedge \forall z[z \leq x \rightarrow \exists z'[P(z') \wedge z \otimes z']]$

  - A sum of a set P is a thing that contains everything in *P* and whose parts each overlap with something in P.
  - "sum(x,P)" means "x is a sum of (the things in) P".

Tarski (1956) (see Betti and Loeb 2012 "On Tarski's Foundations of the Geometry of Solids", *The Bulletin of Symbolic Logic*):

"Definition I. An individual *X* is called a proper part of an individual *Y* if *X* is a part of *Y* and *X* is not identical with *Y*.

Definition II. An individual *X* is said to be disjoint from an individual *Y* if no individual *Z* is part of both *X* and *Y* .

Definition III. An individual *X* is called a sum of all elements of a class α of individuals if every element of α is a part of *X* and if no part of *X* is disjoint from all elements of α. ([Tarski, 1956a], p. 25)

PostulateI. If*X* isapartof*Y* and*Y* isapartof*Z*,then*X* isa part of *Z*.

Postulate II. For every non-empty class α of individuals there ex- ists exactly one individual *X* which is the sum of all elements of α.([Tarski, 1956a], p. 25)"

- The sum operation ' $\oplus$ ' as the primitive notion and the part relation ' $\leq$ ' defined from it.

Krifka (1998, p.199): Definition of a part structure *P*

$P = \langle U_P, \oplus_P, \leq_P, <_P, \otimes_P \rangle$ is a part structure, iff

a. 'U_P' is a set of entities: individuals, eventualities and times

$$I_P \cup E_P \cup T_P \subset U_P$$

b. '$\oplus_P$' is a binary **sum operation**, it is a function from $U_P \times U_P$ to U_P.

It is idempotent, commutative, associative:

$\forall x,y,z \in U_P[x \oplus_P x = x \wedge x \oplus_P y = y \oplus_P x \wedge x \oplus_P (y \oplus_P z) = (x \oplus_P y) \oplus_P z]$

c. '$\leq_P$' is the **part relation**: $\forall x,y \in U_P\ [x \leq_P y \leftrightarrow x \oplus_P y = y]$

d. '$<_P$' is the **proper part relation**: $\forall x,y \in U_P\ [x <_P y \leftrightarrow x \leq_P y \wedge x \neq y]$

e. '$\otimes_P$' is the **overlap relation**: $\forall x,y,z \in U_P\ [x \otimes_P y \leftrightarrow \exists z \in U_P[z \leq_P x \wedge z \leq_P y]]$

f. **remainder principle**: $\forall x,y,z \in U_P\ [x <_P y \rightarrow \exists! z[\neg\ [z \otimes_P x] \wedge z \oplus_P x = y\ ]]$

- An axiom known as the **REMAINDER PRINCIPLE** or SUPPLEMENTATION is used in order to ensure that the following structures be excluded.

  The object *a* has a solitary proper part *b*: a

  b

- **REMAINDER PRINCIPLE**: $\forall x,y,z \in U_P\ [x <_P y \rightarrow \exists! z[\ \neg\ [z \otimes_P x] \wedge z \oplus_P x = y\ ]]$
  Whenever *x* is a proper part of *y*, there is exactly one "remainder" *z* that does not overlap with *x* such that the sum of *z* and *x* is *y* (Krifka 1998).

  Alternative definition: $\forall x,y,z \in U_P\ [x <_P y \rightarrow \exists z[\neg[z \otimes_P x] \wedge z \leq_P y]$

materialization function *h*

$[\![{}^{m}P]\!] = \{x \in M \mid x \leq \sup h\,[[\![P]\!]]\}$ following Bach 1986, p.12

The supremum function 'sup' applies to the materialized counterpart of *P* (the result of applying the function *h* to the denotation of *P*) to give the sum of the quantities of matter which make up the individuals in the interpretation of *P*.

Link's (1998:27/(59) materialization function ***h***:

$[\![{}^{m}\boldsymbol{P}]\!] = \{x \in D \mid x \leq \mathbf{sup}\ h\ [[\![P]\!]]\}$ Bach 1986, p.12

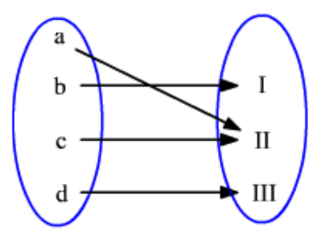
In words: The supremum function 'sup' applies to the materialized counterpart of *P* (the result of applying function *h* to the denotation of *P*) to give the sum of the quantities of matter which make up the individuals in the interpretation of *P*.

Every count predicate ***P*** (*apple* in *There are many/few apples in the salad*) denoting a set of atomic (singular and plural) individuals has a mass term correspondent ***ᵐP*** which denotes a set of quantities of matter in a domain ***D*** (*apple* in *There is much/little apple in the salad*).

$$\boldsymbol{P} \xrightarrow{h} {}^{m}\boldsymbol{P}$$

atomic (sg/pl) individuals stuff making them up

homomorphism: count (domain) non-count (range)



h "is a function (**homomorphism**) from the count elements to the non-count ones, but it is a **many-to-one mapping** so that we can't in general expect a unique answer when we ask what count element this portion of non-count stuff might correspond to" (Bach 1986:11).

Strict Quantization

- STRICTLY_QUANTIZED(P) $=_{def}$ QUANTIZED(P) $\wedge$ $\forall x[P(x) \rightarrow \exists y[y < x]]$ Krifka 1986, 1989
  A predicate *P* is STRICTLY quantized if and only if it has a quantized reference,
  and the entities in *P* are not atoms relative to the whole domain.

  Example: *chair*
  The chair in the picture has a seat and four legs as its proper parts. The seat and the four legs are separate entities relative to the whole domain.
  Chair still denotes a quantized predicate, because no proper part of a chair (e.g., the seat alone) falls under the denotation of chair.

Link (1983) materialization function *h*

(1) a. the cards
  b. the deck of cards

In many contexts, both (1a) and (1b) may refer to the pure collection of objects, and are equivalent in that they are interchangeable; but this does not make them coreferential, i.e., substitutable salva veritate in all the contexts (that are not opaque). The collective term *deck* in (1b) indicates reference to a different individual than in (1a).

(2) Take for *a, b* two rings recently made out of some old Egyptian gold. Then *the rings, a*$\oplus$*b,* are new, *the stuff, a+b,* is old.

(3) A committee is not just the collection of its members, etc. There might be two different committees which necessarily consist of exactly the same members. ("Note, by the way, that the transition to an intension function would be of no help here.")

The Principle of Identity of Indiscernibles or Leibniz's Law (Leibniz *Discourse on Metaphysics,* Section 9):

- an ontological principle that states that two or more entities are identical (are one and the same entity), if they have all their properties in common; or no two objects have exactly the same properties.
- is motivated by the intuition that all we can know of objects, all that is physically relevant about objects, is the properties they instantiate; this would seem to be plausible given that objects' properties determine how they behave, and thus how we can interact with them. The substitution of one object for a 'distinct' yet indiscernible object would be just that — indiscernible — and could make no practical difference.

Application to language:

- Concepts (expressed by referential noun phrases) and thoughts (expressed by sentences) are the same if they play the same role.
- Concepts and thoughts 'play the same role', if the words that express them are interchangeable without affecting the truth-value of the propositions in which they occur: substitutione salva veritate - one of the most basic principles of Leibniz's logic and philosophy of language, which provides a link between the concepts of IDENTITY and TRUTH (CONDITIONS).