Description Logics: a Nice Family of Logics — Introduction, Part 1 —

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Welcome!

Let us know if you

... have questions. – **Do ask** them at any time.

... have difficulties understanding us/reading our writing/...

In this course, we'll

... ask you to **think** a lot

... ask you to **work** through numerous examples

... talk about **complex** stuff with many **fascinating** facets!

What's in this course?

Mon **Introduction**
- Origins, the basic DL $\mathcal{ALC}$, reasoning problems
- Ontologies, examples and exercises

Tue
- Relation with other logics

Tableau algorithms
- for $\mathcal{ALC}$

Wed
- for extensions of $\mathcal{ALC}$

Complexity of selected DLs
- for $\mathcal{ALC}$ and extensions

Thu
- with restrictions
- (un)decidability border

Fri **Further reasoning problems**
- modularity
- justifications

Plan for today

1. Origins of DLs
2. DL basics
3. Ontologies, OWL, Protégé $\rightsquigarrow$ Uli

And now . . .

1. Origins of DLs
2. DL basics
3. Ontologies, OWL, Protégé ⇝ Uli

DLs: where they come from

DLs as **knowledge representation (KR)** formalisms

- Common perception: **logic** is difficult for human conception
  - e.g., how long does it take you to read

$$\forall x \exists y \forall z ((r(x,y) \wedge s(y,z)) \Rightarrow (\neg s(a,y) \vee r(x,z)))$$

  - or check that it is equivalent to

$$\forall x \exists y \forall z (r(x,z) \vee \neg r(x,y) \vee \neg s(y,z) \vee \neg s(a,y))$$

⇝ It's like a new language to learn!
Only for the "mathematically gifted"

- Are there better suited alternatives?
- Can we help users learn/speak/interact with logic?
  They might not even have to see it.

Early KR formalisms

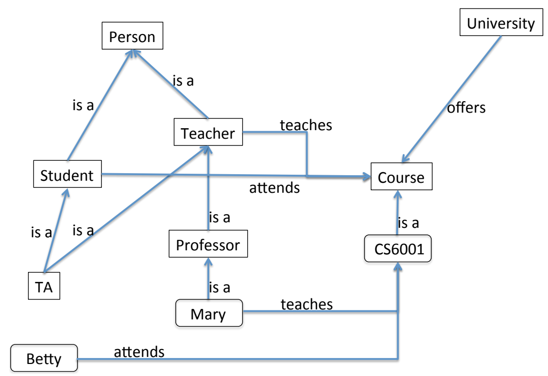
. . . were mostly **graphical** because graphics are

- easier to grasp:
  "A picture says more than a thousand words."
- close to how knowledge is represented in human beings (?)

Most graphical KR formalisms represent knowledge as **graphs** with

- vertices (possibly labelled)
  mostly representing concepts, classes, individuals etc.
- edges (possibly labelled)
  mostly representing properties, relationships etc.

A Semantic Network



What does it represent/say? Is Betty a Student?

Problem: missing semantics
Remedy: base your picture on logic or use logic directly

Terminological Knowledge

DLs: designed to represent **terminological** or **conceptual knowledge**

Goal

- Formalise basic terminology of an application domain; store it in a **TBox**
- Enable reasoning about **concepts** – e.g.:
  - Can there be Mammals?
  - Is every Mammal an Animal?
  - Are Frogs Reptiles?
- Store facts about individuals in an **ABox**
- Enable reasoning about **individuals** and **concepts** – e.g.:
  - Are my facts consistent with my terminology?
  - Is Kermit a Frog?

Applications

Medical

- **SNOMED CT**
  (Systematized Nomenclature of Medicine – Clinical Terms)
  - clinical terminology, used internationally
  - 320,000 terms
- **NCI Thesaurus** (NCI = National Cancer Institute of the USA)
  - vocabulary for clinical care, translational and basic research, public information, administrative activities
  - 120,000 terms
- **ICD 11** (International Classification of Diseases)
  used worldwide for health statistics

Applications

Biology

- **GO** (Gene Ontology)
  controlled vocabulary of terms for gene product characteristics and gene product annotation data
- **Bioportal**
  website that provides access to 530 bio-health ontologies

Semantic Web

- Use terms defined in a TBox to annotate (linked open) data
- Use TBox when querying data

And now . . .

1. Origins of DLs
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DLs: the core

Core part of a DL: its **concept language**, e.g.:

$$\texttt{Animal} \sqcap \exists \texttt{hasPart.Feather}$$

describes all animals that are related via "hasPart" to a feather.

Syntactic ingredients of a concept language:

- **Concept names stand for sets of elements**, e.g., Animal
- **Role names** stand for binary relations between elements, e.g., hasPart
- **Constructors** to build **concept expressions**, e.g., $\sqcap$, $\exists$

Syntax and semantics of $\mathcal{ALC}$

Semantics given by means of an **interpretation** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where

- $\Delta^{\mathcal{I}}$ is a nonempty set (the **domain**), and
- $\cdot^{\mathcal{I}}$ is a mapping (the **interpretation function**) as follows:

Constructor	Syntax	Example	Semantics
concept name	A	Human	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
role name	r	likes	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

For C, D concepts and r a role name:

conjunction	$C \sqcap D$	Human $\sqcap$ Male	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
disjunction	$C \sqcup D$	Nice $\sqcup$ Rich	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
negation	$\neg C$	$\neg$Meat	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
restrictions:			
existential	$\exists r.C$	$\exists$hasChild.Human	$\{x \mid \exists y.(x,y) \in r^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
value	$\forall r.C$	$\forall$hasChild.Blond	$\{x \mid \forall y.(x,y) \in r^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\}$

Understanding syntax and semantics of $\mathcal{ALC}$

We can "draw" interpretations ...
(similarly to Kripke models if you happen to know modal logic)

Exercise 1: Formulate $\mathcal{ALC}$ concepts that describe

1. happy pet owners
2. unhappy pet owners who own an old cat
3. pet owners who own a cat, a dog, and only cats and dogs
4. pet owners who own a cat, a dog, and no other animals
5. everything (abbreviated by $\top$ with $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$)
6. nothing (abbreviated by $\bot$ with $\bot^{\mathcal{I}} = \emptyset^{\mathcal{I}}$)

For each of your concepts (1)–(4),
"draw" an interpretation with an instance of that concept.

Basic reasoning problems in $\mathcal{ALC}$

Definition: let C, D be $\mathcal{ALC}$ concepts. We say that

- $e \in C^{\mathcal{I}}$ is **an instance of** C in $\mathcal{I}$.
- C is **satisfiable** if there is an interpretation $\mathcal{I}$ with $C^{\mathcal{I}} \neq \emptyset$.
- C is **subsumed by** D (written $\models C \sqsubseteq D$) if: for every interpretation $\mathcal{I}$, we have that $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$.

Exercise 2: Which of the following concepts is satisfiable? Which is subsumed by which?

(1) $\exists r.(A \sqcap B)$ (2) $\exists r.(A \sqcup B)$
(3) $\forall r.(A \sqcap B)$ (4) $\exists r.(A \sqcap \neg A)$

(5) $\exists r.A \sqcap \forall r.B$ (6) $\exists r.A$
(7) $\exists r.A \sqcap \forall r.\neg A$ (8) $\exists r.A \sqcap \forall s.\neg A$

The TBox

Definition

- A **general concept inclusion (GCI)** has the form $C \sqsubseteq D$, for C, D (possibly complex) concepts
- A **general TBox** is a finite set of GCIs: $\mathcal{T} = \{C_i \sqsubseteq D_i \mid 1 \leqslant i \leqslant n\}$
- $\mathcal{I}$ **satisfies** $C \sqsubseteq D$ if $C^\mathcal{I} \subseteq D^\mathcal{I}$ (written $\mathcal{I} \models C \sqsubseteq D$)
- $\mathcal{I}$ is a **model of TBox** $\mathcal{T}$ if $\mathcal{I}$ satisfies every $C_i \sqsubseteq D_i \in \mathcal{T}$
- We use $C \equiv D$ to abbreviate $C \sqsubseteq D$, $D \sqsubseteq C$

Example:

$$\begin{aligned} \{\ \texttt{Father} &\equiv \texttt{Man} \sqcap \exists\,\texttt{hasChild.Human}\,, \\ \texttt{Human} &\equiv \texttt{Mammal} \sqcap \forall\,\texttt{hasParent.Human}\,, \\ \exists\,\texttt{favourite.Brewery} &\sqsubseteq \exists\,\texttt{drinks.Beer}\ \} \end{aligned}$$

Exercise 3: Draw a model of the above TBox.
Draw an interpretation that is **not** a model of it.

Reasoning problems with respect to a TBox

Definition: let C, D be concepts, $\mathcal{T}$ a TBox. We say that

- C is **satisfiable w.r.t.** $\mathcal{T}$ if there is a model $\mathcal{I}$ of $\mathcal{T}$ with $C^\mathcal{I} \neq \emptyset$
- C is **subsumed by** D **w.r.t.** $\mathcal{T}$ (written $\mathcal{T} \models C \sqsubseteq D$) if, for every model $\mathcal{I}$ of $\mathcal{T}$, we have $C^\mathcal{I} \subseteq D^\mathcal{I}$

Example:

$$\begin{aligned} \mathcal{T} = \{ \quad A &\sqsubseteq B \sqcap \exists r.C\,, \\ \exists r.\top &\sqsubseteq \neg A \qquad \} \end{aligned}$$

Exercise 4: Does $\mathcal{T}$ have a model?
Are all concept names in $\mathcal{T}$ satisfiable?
Any subsumptions that you can point out?

How many models does a TBox have?

The ABox

TBox
- captures knowledge on a general, conceptual level
- contains concept def.s + general axioms about concepts

ABox
- captures knowledge on an the level of individuals
- is a finite set of
  - **concept assertions** $a : C$ e.g., John : Man, and
  - **role assertions** $(a, b) : r$ e.g., (John, Mary) : hasChild

Semantics: an interpretation $\mathcal{I}$

- maps each **individual name** e to some $e^\mathcal{I} \in \Delta^\mathcal{I}$
- satisfies a concept assertion $a : C$ if $a^\mathcal{I} \in C^\mathcal{I}$
- satisfies a role assertion $(a, b) : r$ if $(a^\mathcal{I}, b^\mathcal{I}) \in r^\mathcal{I}$
- is a **model** of an ABox $\mathcal{A}$ if $\mathcal{I}$ satisfies every assertion in $\mathcal{A}$

$a : C$ is **entailed by** $\mathcal{A}$ if every model of $\mathcal{A}$ satisfies $a : C$

The ABox

repeated from previous slide

Semantics: an interpretation $\mathcal{I}$

- maps each **individual name** e to some $e^\mathcal{I} \in \Delta^\mathcal{I}$
- satisfies a concept assertion $a : C$ if $a^\mathcal{I} \in C^\mathcal{I}$
- satisfies a role assertion $(a, b) : r$ if $(a^\mathcal{I}, b^\mathcal{I}) \in r^\mathcal{I}$
- is a **model** of an ABox $\mathcal{A}$ if $\mathcal{I}$ satisfies every assertion in $\mathcal{A}$

$a : C$ is **entailed by** $\mathcal{A}$ if every model of $\mathcal{A}$ satisfies $a : C$

Example:

$$\begin{aligned} \mathcal{A} = \{ \quad & a : (B \sqcap \exists r.C)\,, \\ & b : (A \sqcap \neg P \sqcap \forall s.\forall r.F)\,, \\ & (b, a) : s\ \} \end{aligned}$$

Exercise 5: Does $\mathcal{A}$ have a model? – Describe some of them.
Can you see any entailments?

(Later) Can you translate this into FOL? ML?

Ontologies: TBox and ABox

Definition: an **ontology** consists of

- a TBox that captures knowledge on a general, conceptual level
- an ABox that captures knowledge on the level of individuals and **uses terms described in the TBox**

Notation: $(\mathcal{T}, \mathcal{A})$ or $\mathcal{T} \cup \mathcal{A}$ – no difference!

Semantics:

- Int. $\mathcal{I}$ is a **model** of $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ (written $\mathcal{I} \models \mathcal{O}$) if $\mathcal{I}$ satisfies every assertion and axiom in $\mathcal{O}$
  alternatively: $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \models \mathcal{A}$
- $\mathcal{O}$ is **consistent** if it has a model
- $\mathcal{O}$ is **coherent** if every conc. name A in $\mathcal{O}$ is satisfiable w.r.t. $\mathcal{O}$
- $C \sqsubseteq D$ is **entailed by** $\mathcal{O}$ if every model of $\mathcal{O}$ satisfies $C^\mathcal{I} \subseteq D^\mathcal{I}$
- $a : C$ is **entailed by** $\mathcal{O}$ if every model of $\mathcal{O}$ satisfies $a^\mathcal{I} \in C^\mathcal{I}$

Ontologies: TBox and ABox

Semantics: *repeated from previous slide*

- Int. $\mathcal{I}$ is a **model** of $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ if $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \models \mathcal{A}$
- $\mathcal{O}$ is **consistent** if it has a model
- $\mathcal{O}$ is **coherent** if every conc. name A in $\mathcal{O}$ is satisfiable w.r.t. $\mathcal{O}$
- $C \sqsubseteq D$ is **entailed by** $\mathcal{O}$ if every model of $\mathcal{O}$ satisfies $C^\mathcal{I} \subseteq D^\mathcal{I}$
- $a : C$ is **entailed by** $\mathcal{O}$ if every model of $\mathcal{O}$ satisfies $a^\mathcal{I} \in C^\mathcal{I}$

Example:

$$\mathcal{O} = \{ \quad A \sqsubseteq B \sqcap \exists r.C, \quad a : B, \quad \exists r.\top \sqsubseteq \neg A, \quad (a,b) : r \quad \}$$

Exercise 6: Does $\mathcal{O}$ have a model? – Describe some of them.
Can you see any entailments?
What about $\mathcal{O} \cup \{b : C\}$ or $\mathcal{O} \cup \{b : A\}$?

Ontologies: TBox and ABox

Semantics: *repeated from previous slide*

- Int. $\mathcal{I}$ is a **model** of $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ if $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \models \mathcal{A}$
- $\mathcal{O}$ is **consistent** if it has a model
- $\mathcal{O}$ is **coherent** if every conc. name A in $\mathcal{O}$ is satisfiable w.r.t. $\mathcal{O}$
- $C \sqsubseteq D$ is **entailed by** $\mathcal{O}$ if every model of $\mathcal{O}$ satisfies $C^\mathcal{I} \subseteq D^\mathcal{I}$
- $a : C$ is **entailed by** $\mathcal{O}$ if every model of $\mathcal{O}$ satisfies $a^\mathcal{I} \in C^\mathcal{I}$

Lemma

If $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ is consistent, then:
$C \sqsubseteq D$ is entailed by $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ iff $C \sqsubseteq D$ is entailed by $\mathcal{T}$.

Proof: for "$\Leftarrow$" note that every model of $\mathcal{O}$ is one of $\mathcal{T}$.

For "$\Rightarrow$" use contraposition; combine a model witnessing $\mathcal{T} \not\models C \sqsubseteq D$ and one of $\mathcal{O}$ to one witnessing $\mathcal{O} \not\models C \sqsubseteq D$). □

And now ...

References for Day 1

- Franz Baader, Diego Calvanese, Deborah L. McGuinness, Daniele Nardi, Peter F. Patel-Schneider (editors).
  The Description Logic Handbook:
  Theory, Implementation and Applications.
  2nd edition, Cambridge University Press, 2007.
  ISBN 978-0521876254.
  Chapters 1–2.
- Ian Horrocks, Peter F. Patel-Schneider, Frank van Harmelen.
  From $\mathcal{SHIQ}$ and RDF to OWL:
  the making of a Web Ontology Language.
  Journal of Web Semantics 1(1): 7–26, 2003.

Origins Basics Ontologies, OWL, Protégé

Links for Day 1

- The National Center for Biomedical Ontology:
  BioPortal
  http://bioportal.bioontology.org
- Stanford Center for Biomedical Informatics Research:
  Protégé Ontology Editor
  http://protege.stanford.edu