day 3

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Logical foundations of databases

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Recap

- **Active domain semantics and expressiveness:** $FO^{act} =^{*} RA$

- **Undecidable problems** (Halting ≤ Domino ≤ FO-Satisfiability ≤ FO-Equivalence)

- **Data complexity / Combined complexity**

- **Evaluation problem for FO:** in PSPACE (combined comp.)
  in PSPACE (query comp.)
  in LOGSPACE (data comp.)

- **Positive FO:** evaluation in NP (combined comp.)

- Conjunctive Queries

Conjunctive Queries

Def.

CQ = FO without $\forall, \neg, \vee$

Eg: $\phi(x, y) = \exists z . (\text{Parent}(x, z) \wedge \text{Parent}(z, y))$

Usual notation: "Grandparent(X,Y) : – Parent(X,Z), Parent(Z,Y)"

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quantifier-free and no equalities!

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It corresponds to positive "SELECT-FROM-WHERE" SQL queries

```
Select ...
From ...
Where Z
```

no negation or disjunction

It corresponds to "π-σ-×" RA queries

$\pi_X(\sigma_Z(R_1 \times \cdots \times R_n))$

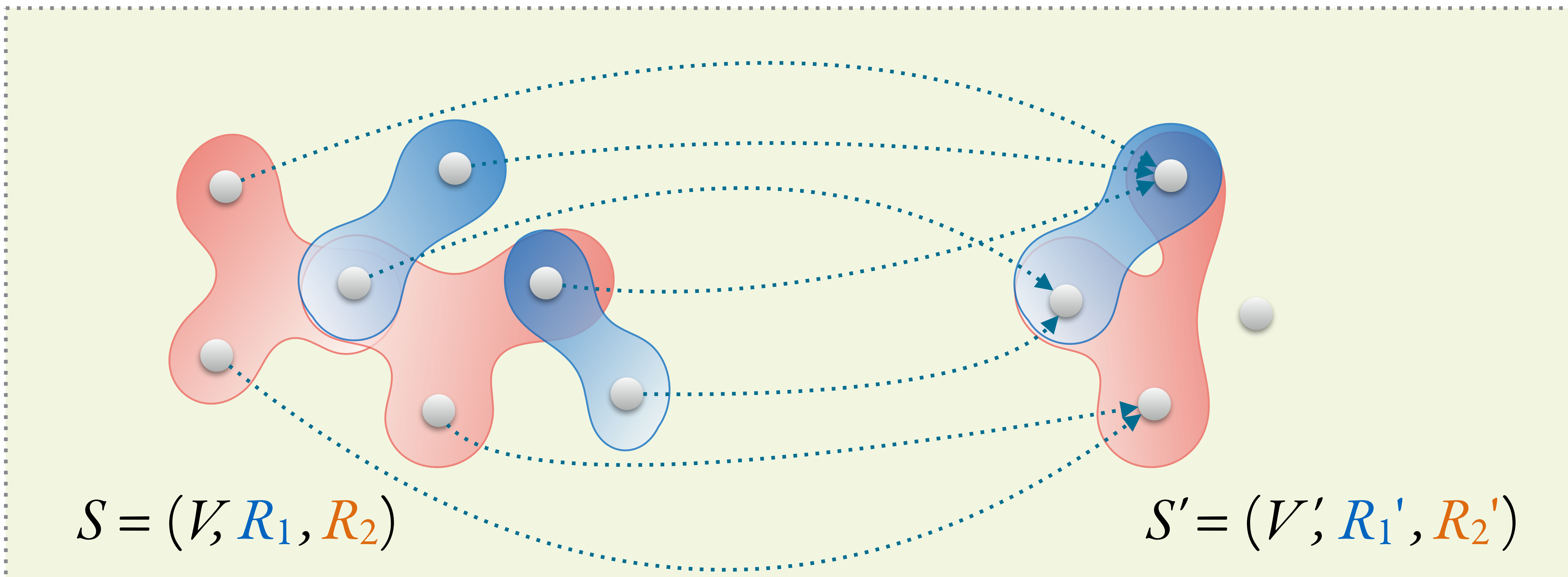
no negation

Homomorphisms

Homomorphism between structures $S=(V,R_1,\ldots,R_n)$ and $S'=(V',R_1',\ldots,R_n')$

is a function $h: V \longrightarrow V'$ such that

$(x_1,\ldots,x_n) \in R_i$ implies $(h(x_1),\ldots,h(x_n)) \in R_i'$



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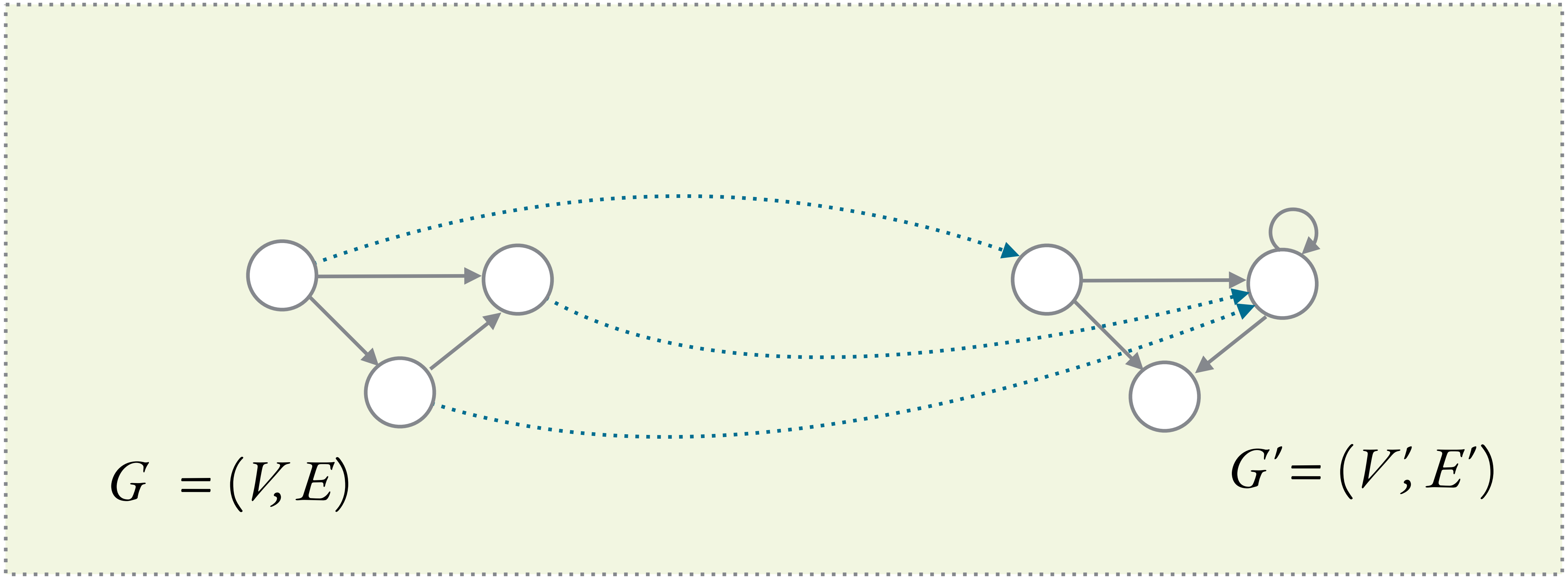
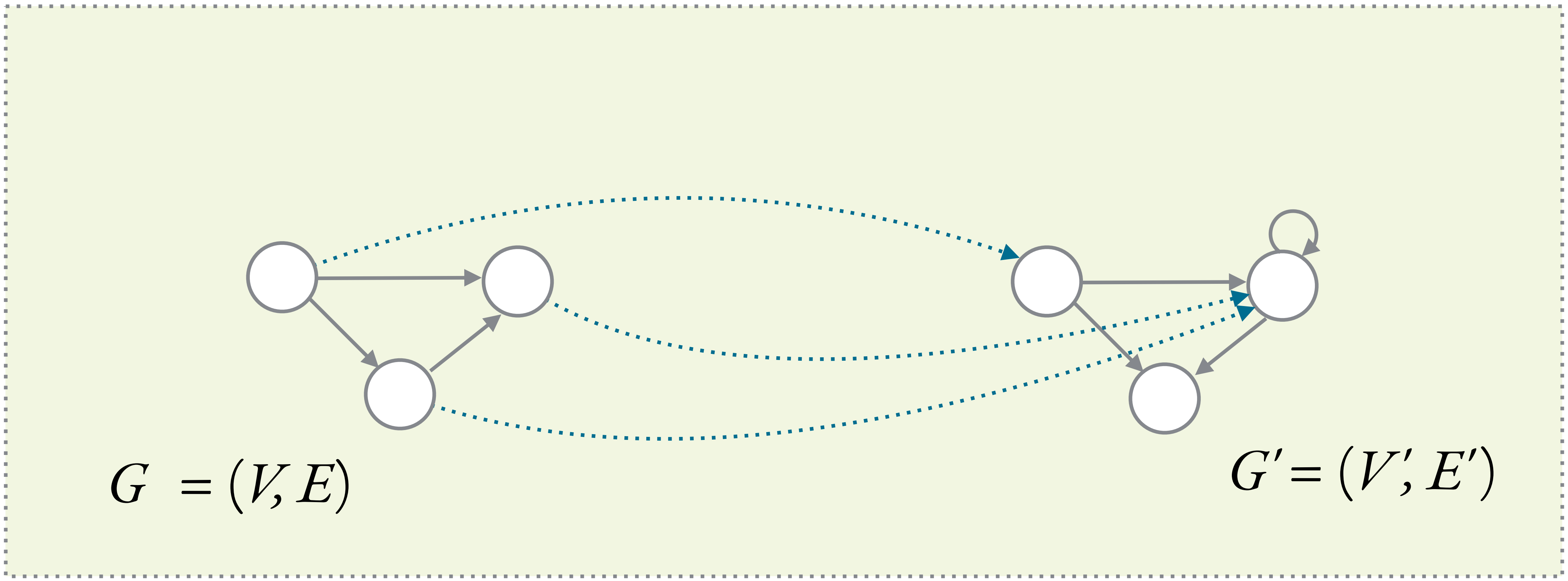


Canonical structures

Canonical structure G_ϕ of a Conjunctive Query ϕ has
- variables as nodes
- tuples $(x_1, \ldots, x_n) \in R_i$ for all atomic sub-formulas $R_i(x_1, \ldots, x_n)$ of ϕ



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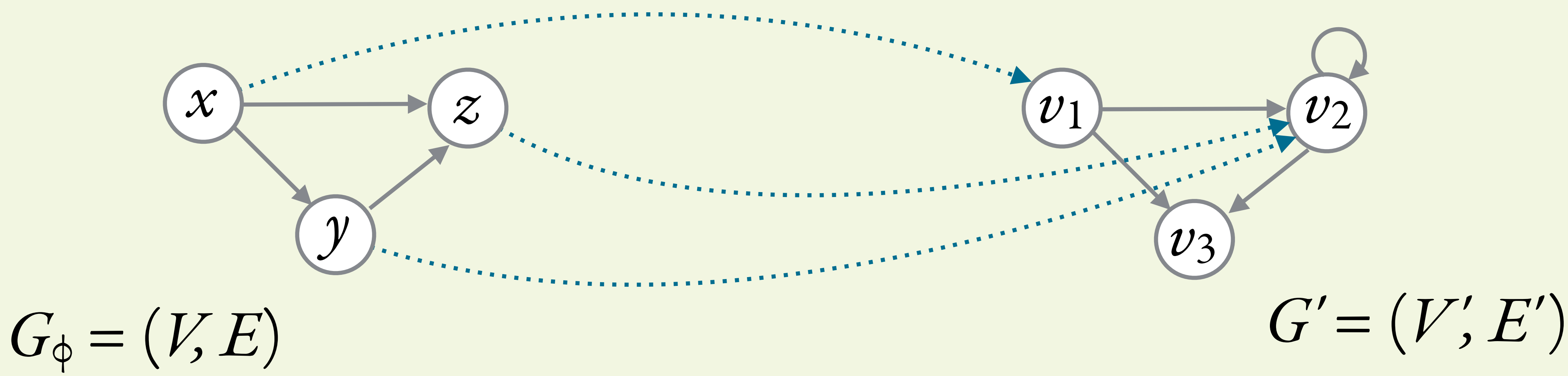

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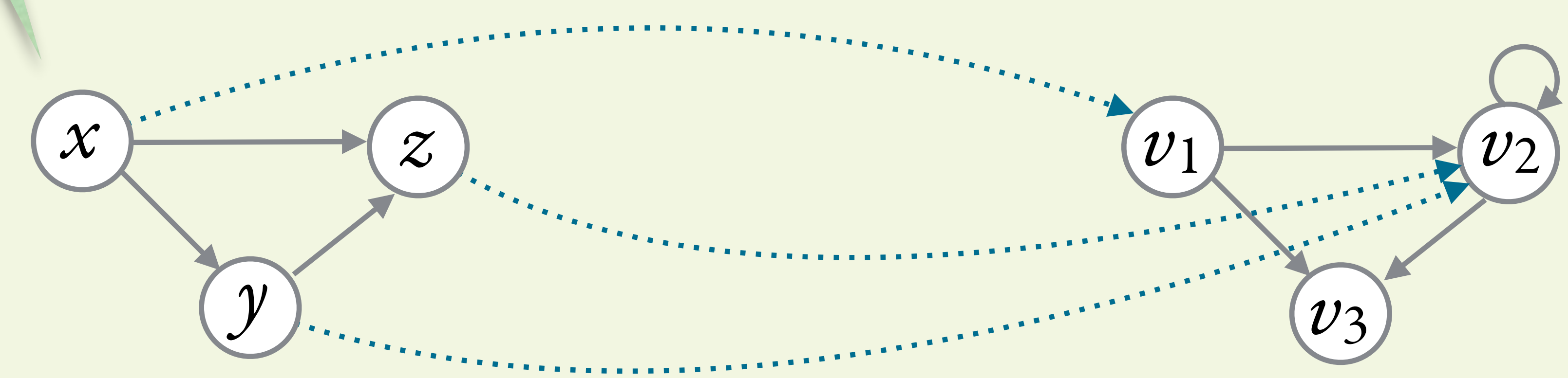

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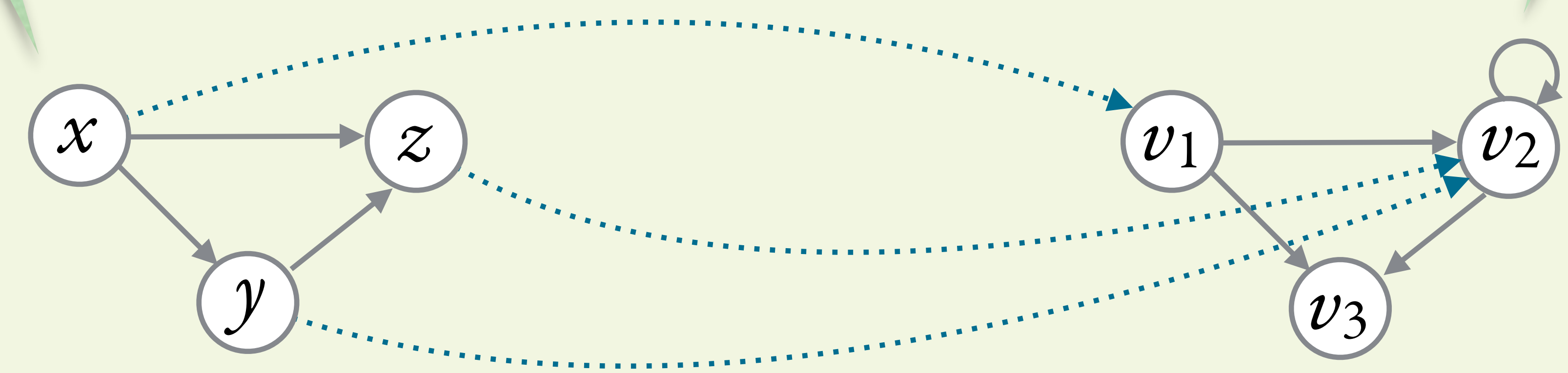

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Fact 3: $G'' \models \phi$ iff $\exists$ h: $G_\phi \longrightarrow G''$

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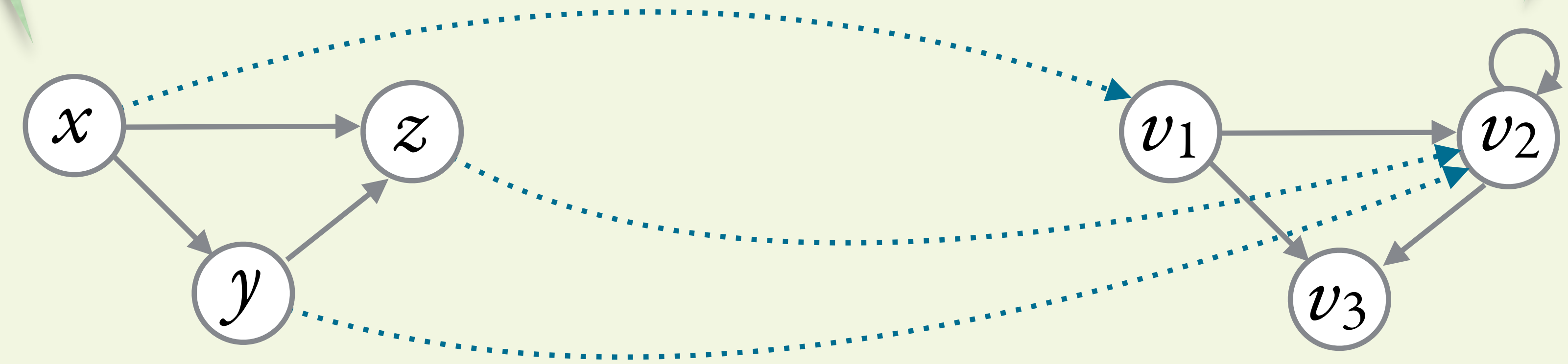

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Evaluation via homomorphisms

Lemma. The evaluation of a CQ $\phi(x_1, \ldots, x_n)$ on S' returns the set

$$\phi(S') = \{ (\mathrm{h}(x_1), \ldots, \mathrm{h}(x_n)) \mid \mathrm{h} : G_\phi \longrightarrow S' \}$$

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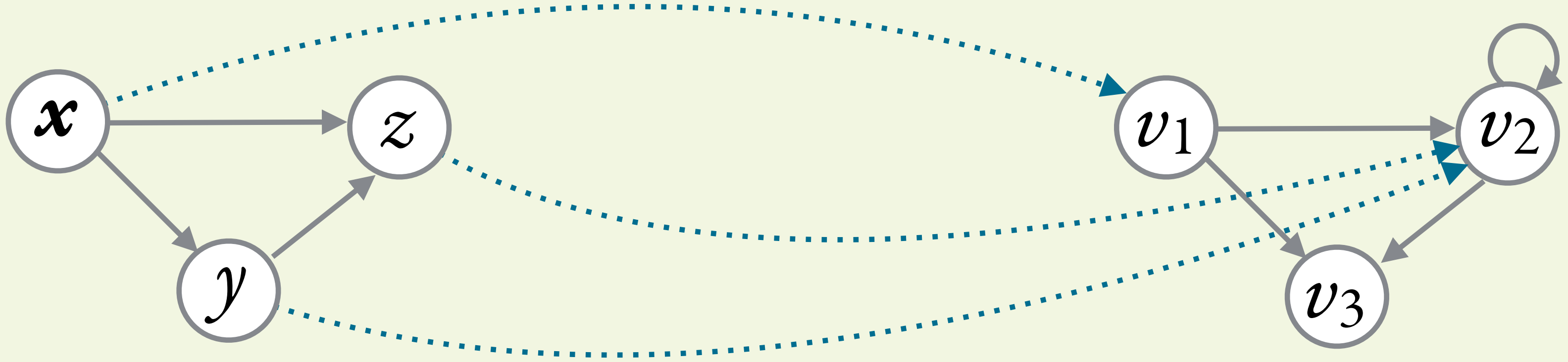

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Question: Which are the homomorphisms $G_\phi \longrightarrow G'$?
What is the result of $\phi(G')$?

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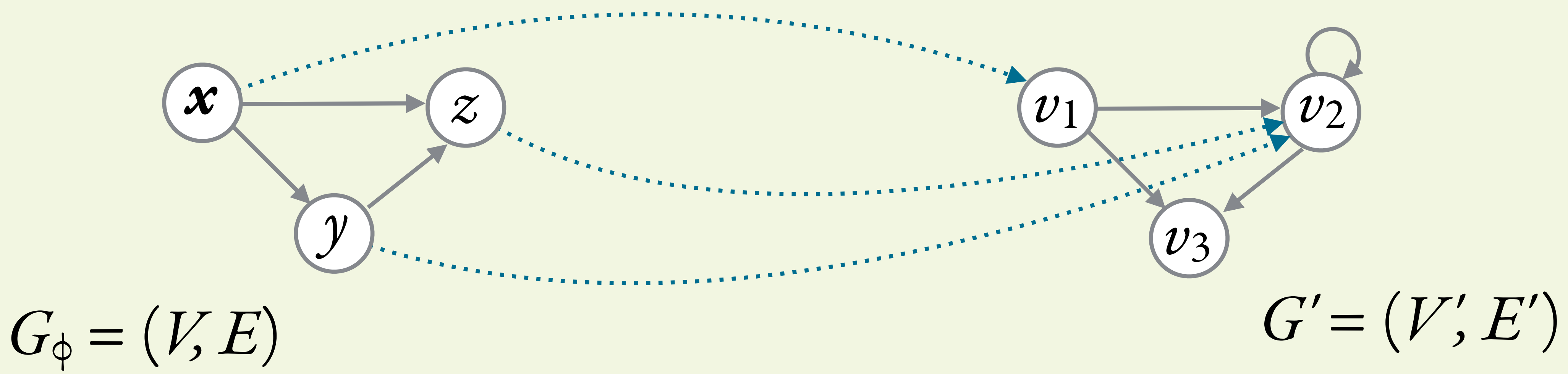
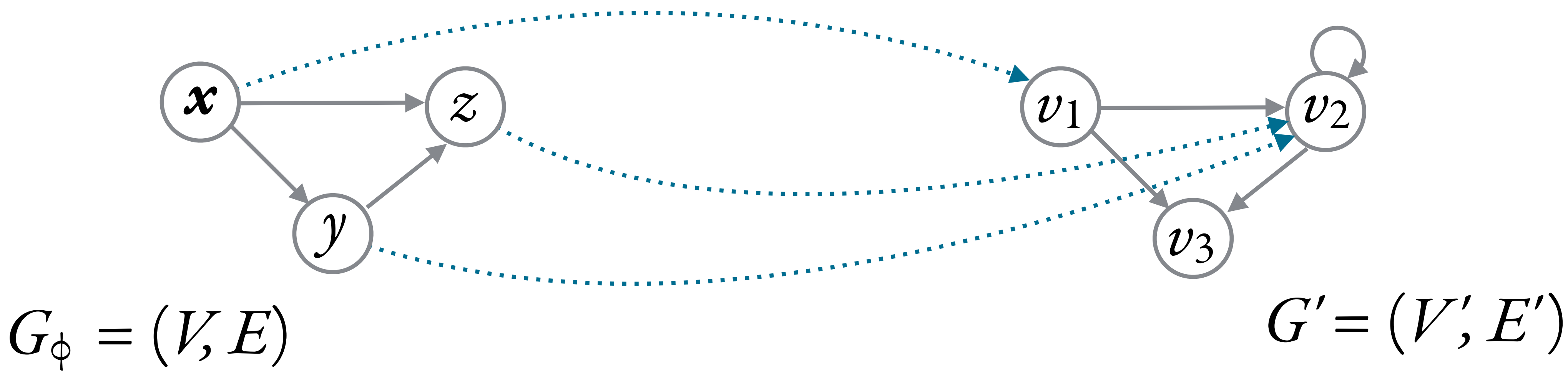


Evaluation via homomorphisms

Theorem. Evaluation of CQ is in NP (combined complexity)



Input: A CQ $\phi(x_1, \ldots, x_n)$, a graph G, a tuple $(a_1, \ldots, a_n)$
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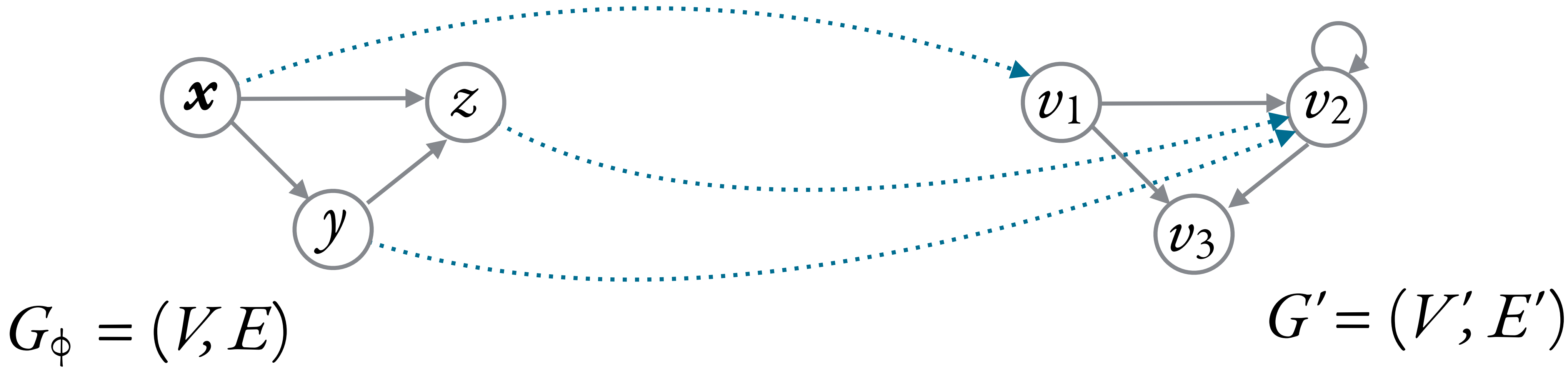

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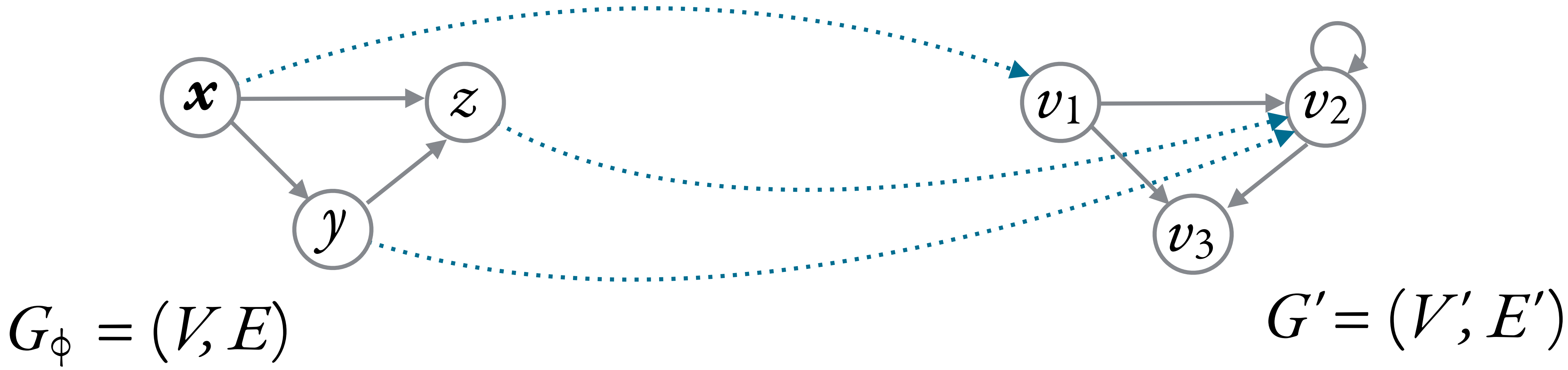

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Ideas?

1. Guess h: $G_\phi \longrightarrow G$
2. Check that it is a homomorphism
3. Output YES if $(h(x_1), \ldots, h(x_n)) = (a_1, \ldots, a_n)$; NO otherwise.

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NP-complete problem: **3-COLORABILITY**

Input: A graph G

Output: Can we assign a colour from $\{R,G,B\}$ to each node so that adjacent nodes have always different colours ?

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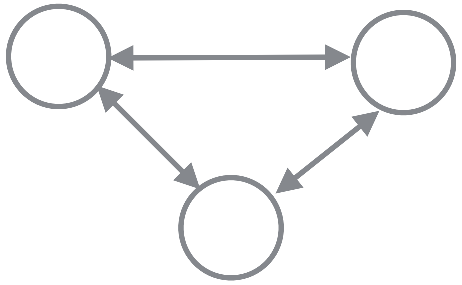
Is there a *homomorphism* from G to K_3 ?

K_3



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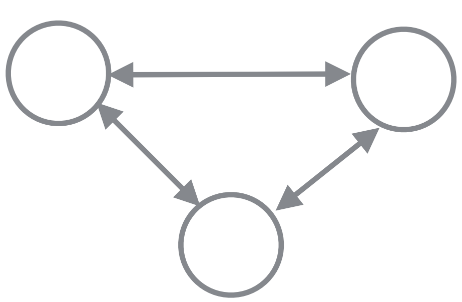

Reduction 3COL ⇝ CQ-EVAL: 1. Given G, build a CQ ϕ such that $G_\phi = G$.
2. Test if $() \in \phi(K_3)$.

Monotonicity and preservation theorems

Lemma. Every CQ is **monotone**:

$S \subseteq S'$ implies $\phi(S) \subseteq \phi(S')$

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Proof:

$$\begin{aligned}\phi(S) &= \{ (h(x_1), ..., h(x_n)) \mid h : G_\phi \longrightarrow S \} \\ &\subseteq \{ (h'(x_1), ..., h'(x_n)) \mid h' : G_\phi \longrightarrow S' \} \\ &= \phi(S')\end{aligned}$$

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$$= \phi(S')$$

- "The relation R has at most 2 elements" $\notin$ CQ
- "There is a node connected to every other node" $\notin$ CQ
- "The radius of the graph is 5" $\notin$ CQ

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Theorem. If an FO query ϕ is **monotone**
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Equally expressive, but
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- One example of the few properties which still hold on finite structures.
- Proof in the finite is difficult and independent.

Static analysis with CQs

The satisfiability problem for CQ is decidable…

Question: What is the algorithm for CQ-SAT? What is the complexity?

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Answer: CQs are always satisfiable by their canonical structure!

$$G_\phi \models \phi$$

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problem: **CQ-CONTAINMENT**

Input: Two CQs ϕ, ψ

Output: Does $\phi(S) \subseteq \psi(S)$ holds for every structure S ?

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$h \circ g$ is a homomorphism from G_ψ to S. Hence, $(v_1, \ldots, v_n) \in \psi(S)$.

Static analysis with CQs

problem: **CQ-EQUIVALENCE**

Input: Two CQs ϕ, ψ

Output: Does $\phi(S) = \psi(S)$ holds for every S ? (we write "$\phi \equiv \psi$")

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2b. There is h: $G_\phi \longrightarrow G_\psi$

3a. $g(y_i) = x_i$ for all i

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Amounts to testing if G_ϕ and G_ψ are **hom-equivalent**
(homomorphisms in both senses)

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Query optimisation: Can I **simplify** the query?

Static analysis with CQs

Query optimisation: Can I **simplify** the query?

problem: **CQ-MINIMIZATION**

Input: A CQ ϕ

Output: Is there a smaller CQ ψ such that $\psi \equiv \phi$?

smaller = with less number of atoms

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Output: Is there a smaller CQ ψ such that $\psi \equiv \phi$?

Theorem. The minimization problem for CQ is NP-complete

Static analysis with CQs

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Amounts to testing if there is a smaller structure hom-equivalent to G_ϕ

≈ testing if there is a **non-injective endomorphism**

$$g \colon G_\phi \longrightarrow G_\phi$$

The **smallest structure hom-equivalent** to S is called the **core** of S, and it is unique.

Static analysis with CQs

Question:
- Is $\phi = \exists\, x,y,z\ \ R(x,y) \wedge R(x,z) \wedge S(z,z) \wedge S(z,y)$ minimal?
- What is its minimal equivalent query?

Answer:

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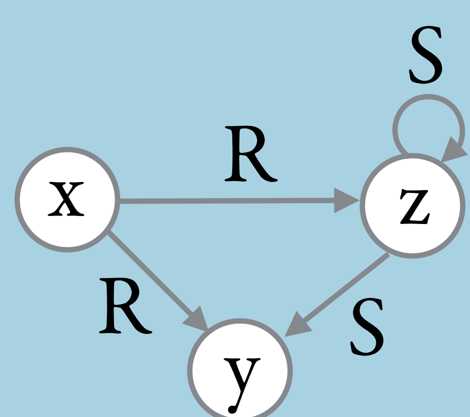

G_ϕ

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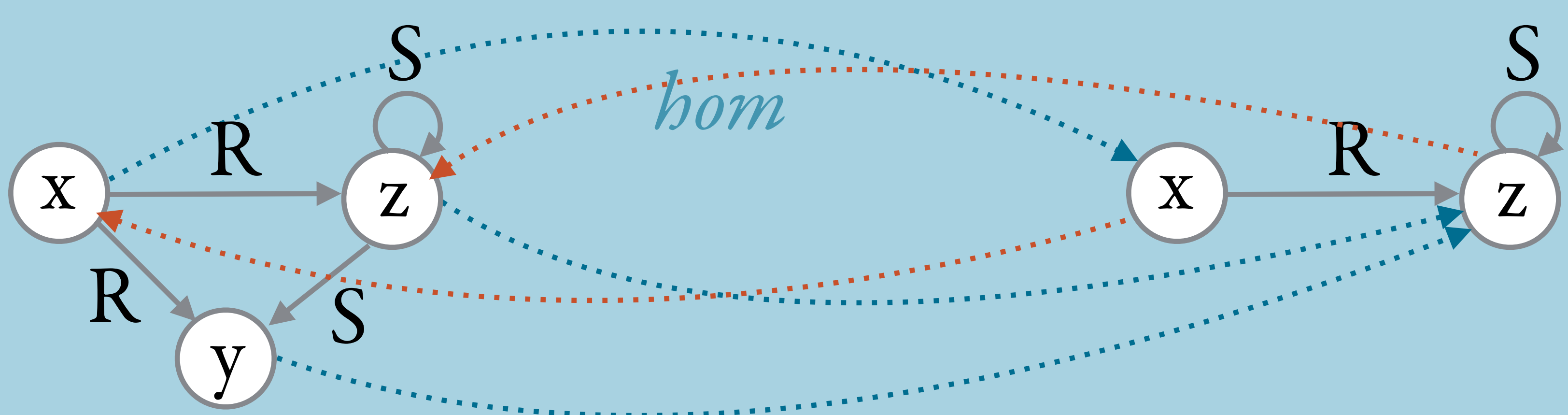

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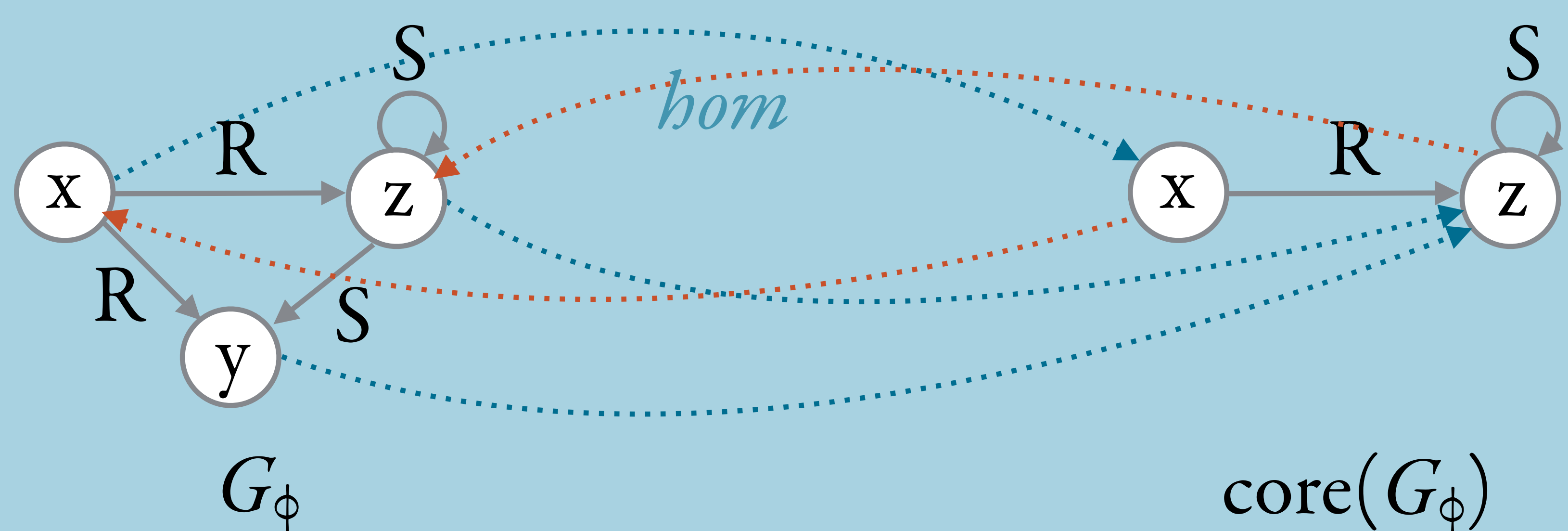

No! $\psi = \exists\, x,z\;\; R(x,z) \wedge S(z,z)$ is the minimal query s.t. $\phi \equiv \psi$

Adding functional dependencies

Adding functional dependencies

e.g.

key constraints like

"column *SSN* determines column *Name* in the table Employees"

(component *i*) (component *j*) (relation)

Adding functional dependencies

A **unary functional dependency** is a sentence of the form

$$\forall x_1,\ldots,x_n,y_1,\ldots,y_n \,.\, R(x_1,\ldots,x_n) \land R(y_1,\ldots,y_n) \land (x_i = y_i) \Rightarrow (x_j = y_j)$$

"**R[i⟶j]**" : in relation R the i-th component determines the j-th component

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Example: In the following relation we may enforce the functional dependency

$$\boldsymbol{\gamma} = \forall x, y, z, x', y', z' \;\; R(x,y,z) \wedge R(x',y',z') \wedge (x = x') \Rightarrow (y = y')$$

Agent	Name	Drives
007	James Bond	Aston Martin
200	Mr Smith	Cadillac
201	Mrs Smith	Mercedes
3	Jason Bourne	BMW

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A structure S **verifies** a set of UFD $\{\phi_1, \ldots, \phi_n\}$ if $S \vDash \phi_1 \land \cdots \land \phi_n$.

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All the previous problems:

- CQ-CONTAINMENT
- CQ-EQUIVALENCE
- CQ-MINIMIZATION

remain in NP if we further restrict finite structures so as to satisfy any set of functional dependencies

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All the previous problems:

Modify the canonical structure G_ϕ ...

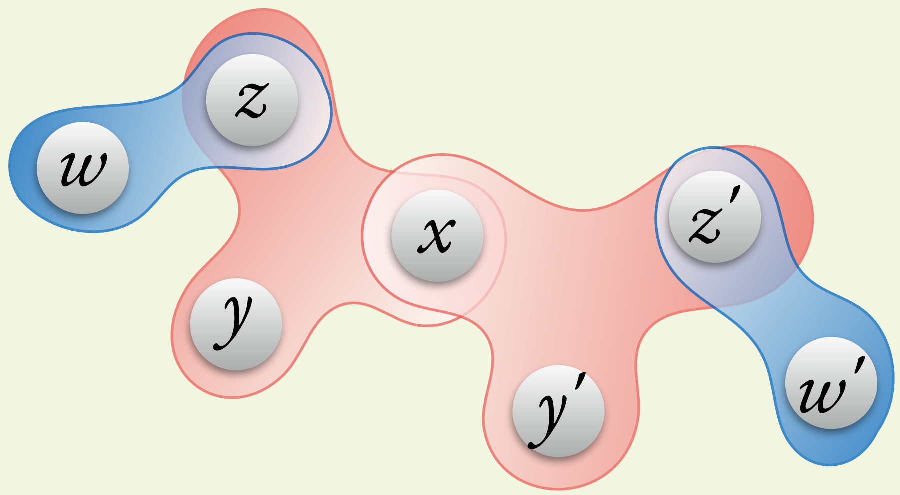
- CQ-CONTAINMENT
- CQ-EQUIVALENCE
- CQ-MINIMIZATION

remain in NP if we further restrict finite structures so as to satisfy any set of functional dependencies

Adding functional dependencies

CQ $\phi = R_2(x, y, z) \wedge R_2(x, y', z') \wedge R_1(z, w) \wedge R_1(z', w')$

under functional dependencies $F=\{\, R_1[1 \longrightarrow 2],\ R_2\!: [1 \longrightarrow 3]\,\}$



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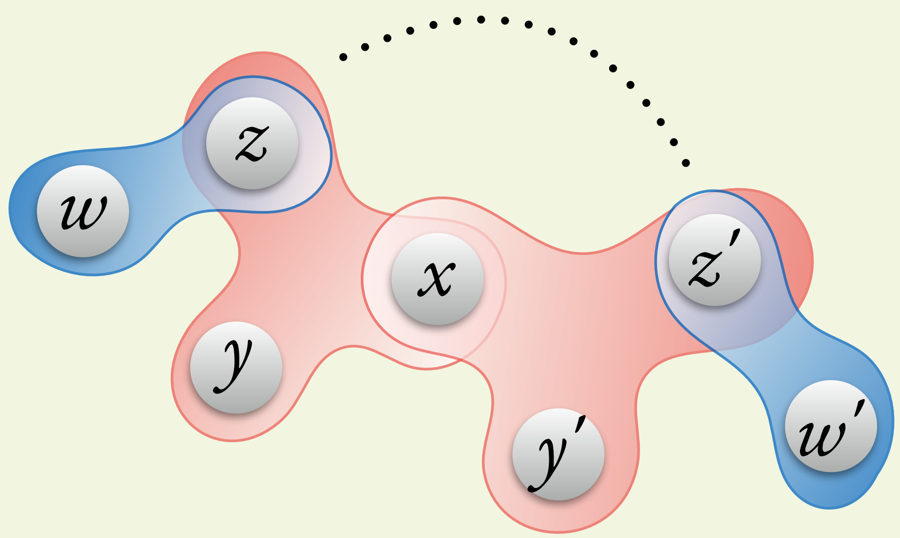

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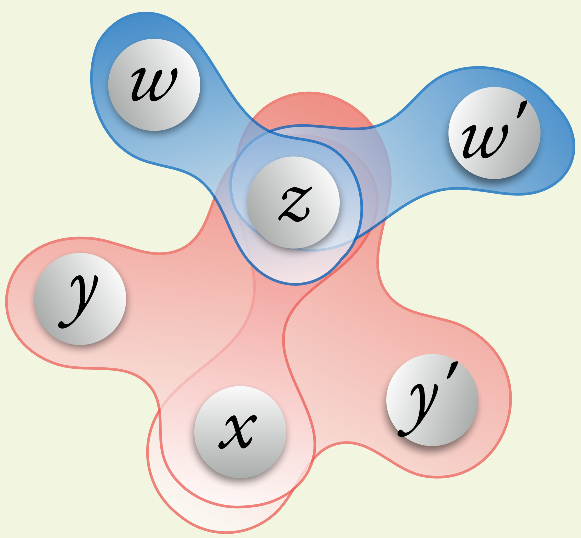

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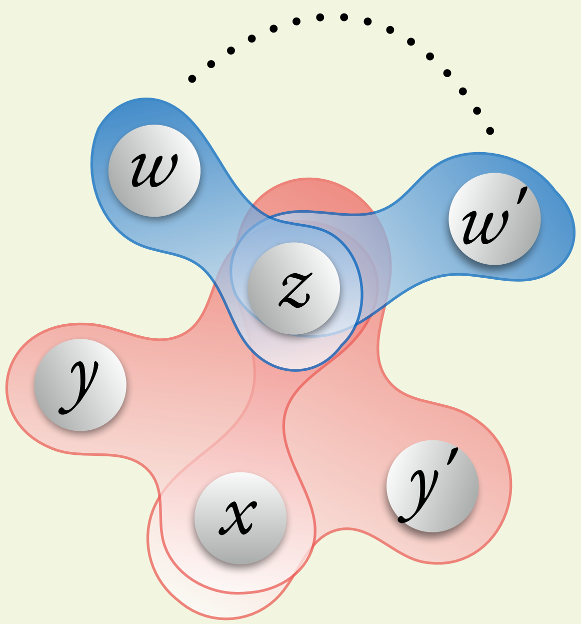

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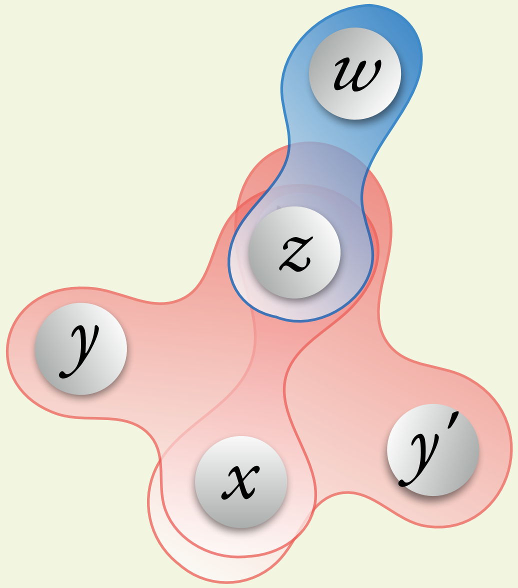

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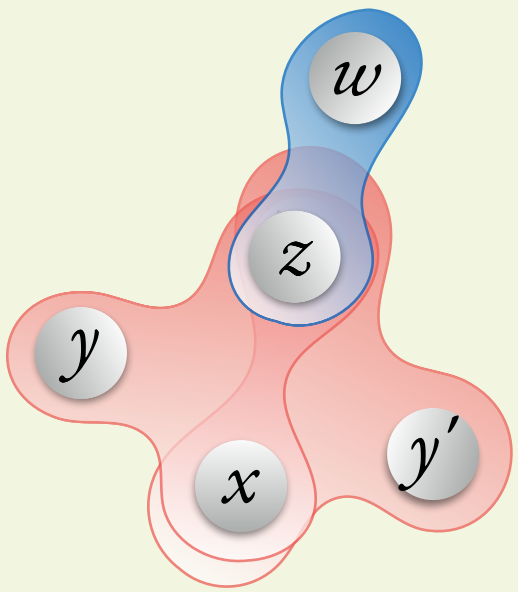

$= \text{chase}_F(G_\phi)$ (the **chased canonical structure**)

- $\text{chase}_F(G_\phi)$ is unique and can be constructed in polynomial time

Adding functional dependencies

$\phi \in CQ$
FD's $F=\{fd_1, \ldots, fd_n\}$

chase →

$chase_F(\phi) \in CQ$

Adding functional dependencies

$\phi \in CQ$
FD's $F=\{fd_1, \ldots, fd_n\}$ —chase→ $chase_F(\phi) \in CQ$

The static analysis problems restricted to FD's can now be also shown in NP

- CQ-Containment: $\phi \subseteq_F \psi$ iff $chase_F(\phi) \subseteq chase_F(\psi)$
- CQ-Equivalence: $\phi \equiv_F \psi$ iff $chase_F(\phi) \equiv chase_F(\psi)$
- CQ-Minimization: ϕ is minimal wrt structures verifying F iff $chase_F(\phi)$ is minimal

Acyclic CQ's : Definition

On graphs: CQ ϕ is **acyclic** if G_{ϕ} is tree-like

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underlying undirected graph is acyclic

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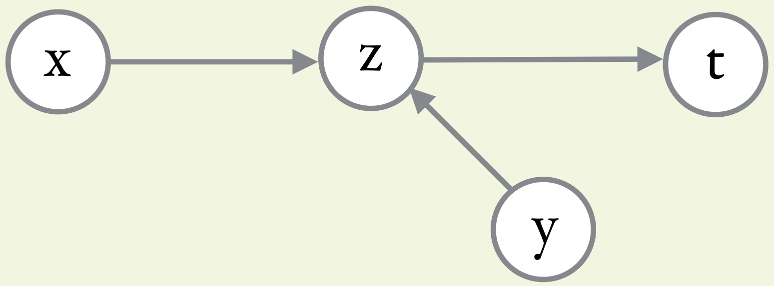
$\phi(x,y) = \exists z \,.\, E(x,z) \wedge E(z,t) \wedge E(y,z)$



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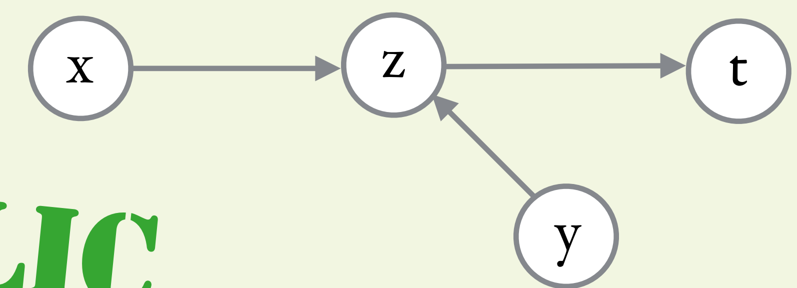
ACYCLIC



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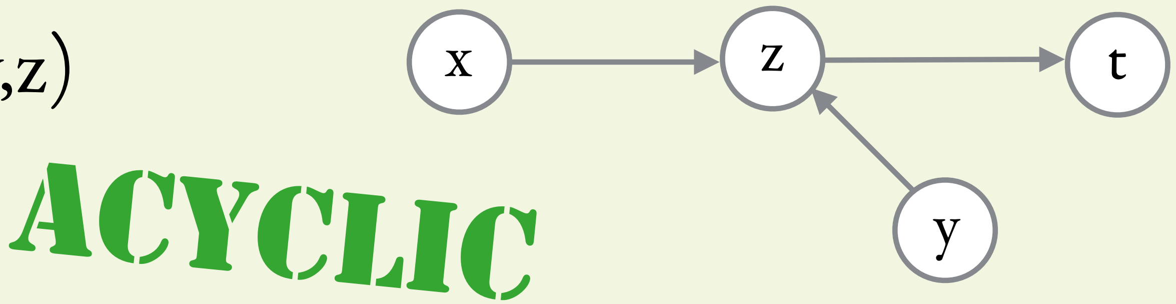

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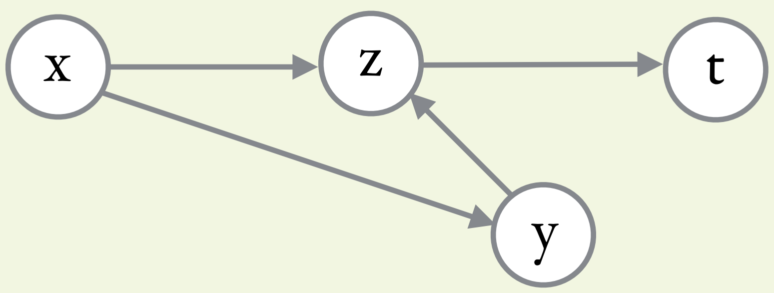

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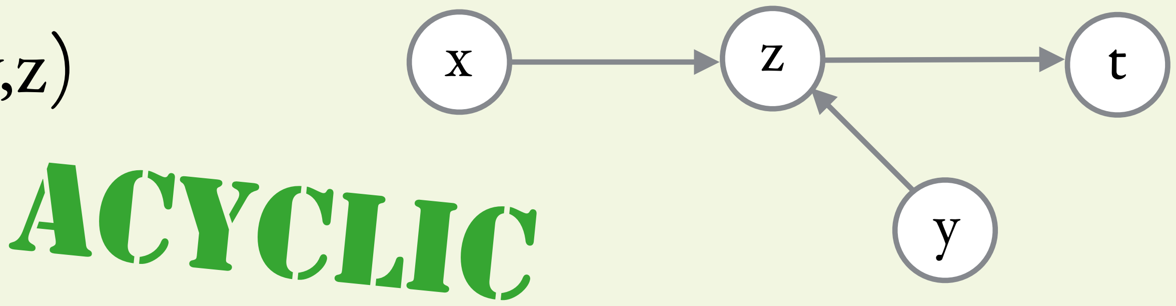

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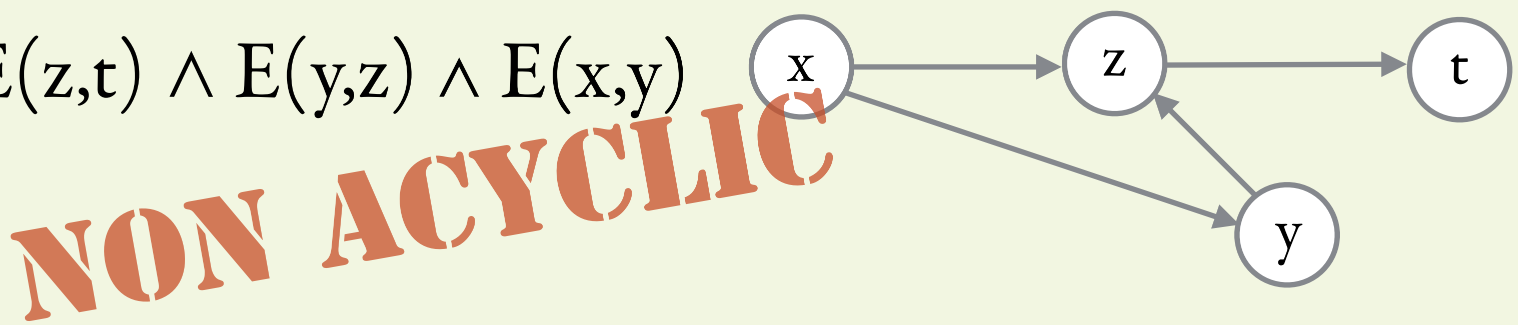

Acyclic CQ's

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Acyclic CQ's

On graphs: CQ ϕ is **acyclic** if G_ϕ is tree-like

On general structures: a CQ ϕ is **acyclic** if it has a join tree

$$\phi(\bar{y}) = \exists \bar{z} \, . \, R_1(\bar{z}_1) \wedge \ldots \wedge R_m(\bar{z}_m)$$

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A **join tree** is a tree T st:

- nodes are the atoms $R_i(\bar{z}_i)$
- for every variable x of ϕ the set of $R_i(\bar{z}_i)$'s with $x \in \bar{z}_i$ forms a subtree of T

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Alternatively, if its canonical hyper-graph is α-acyclic.

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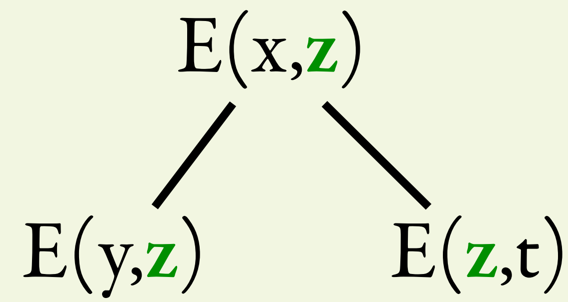
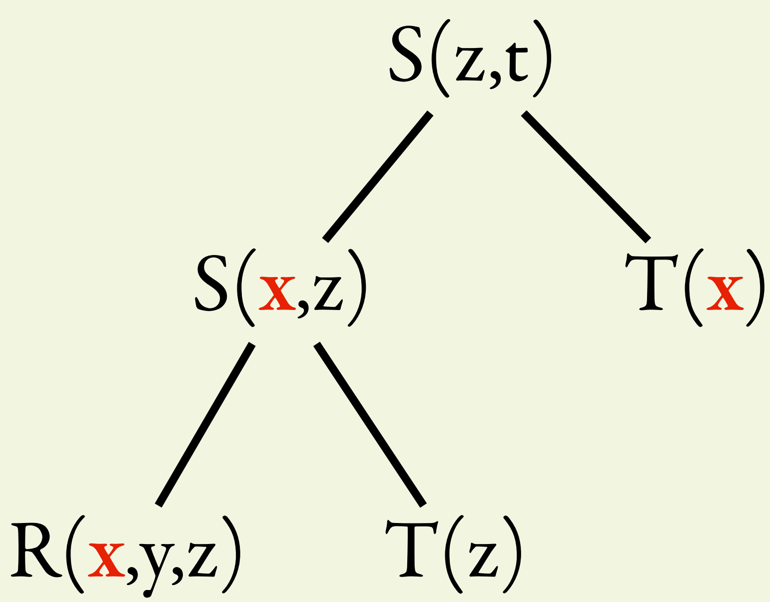
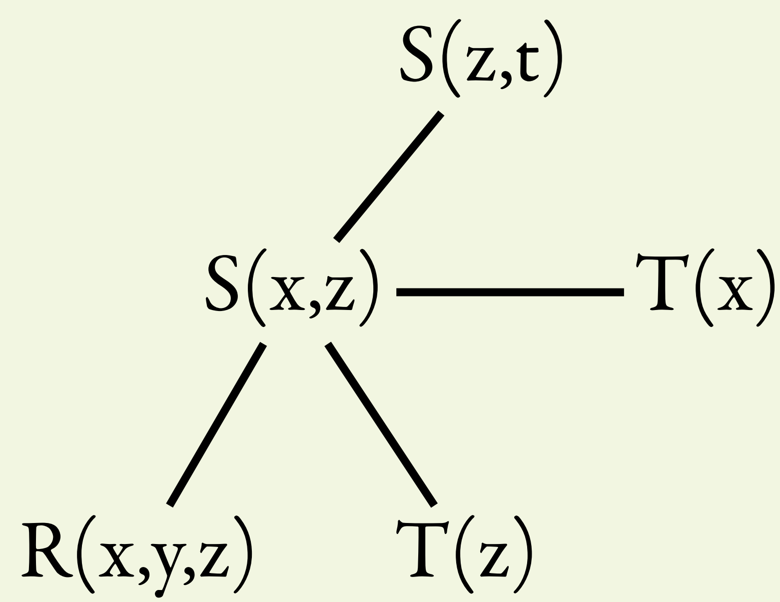


join tree

$\phi = \exists x,y,z,t \,.\, R(x,y,z) \wedge S(z,t) \wedge S(x,z) \wedge T(z) \wedge T(x)$

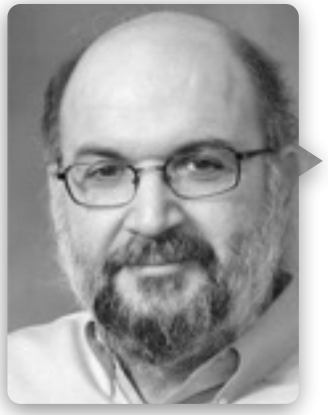


not a join tree



a join tree

Acyclic CQ's



The **evaluation problem** for acyclic CQ sentences is in $O(|\phi|.|D|)$

[Yannakakis]

The **semi-join**

$$R \ltimes_{\{i_1=j_1,\ldots,i_n=j_n\}} S = \{ (x_1,\ldots,x_n) \in R \mid \text{there is } (y_1,\ldots,y_m) \in S \text{ where } x_{i_k} = y_{j_k} \text{ for all } k\}$$

Note: $R \ltimes_{\{i_1=j_1,\ldots,i_n=j_n\}} S \subseteq R$