Introduction to Non(-)monotonic Logic

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ESSLLI 2016, Bolzano

Day 5. Formal argumentation

Dung, Phao Minh. On the acceptability of arguments and its fundamental role in non-monotonic reasoning, logic programming, and *n*-person games. *Artificial Intelligence* 77 (1995): 321–357.

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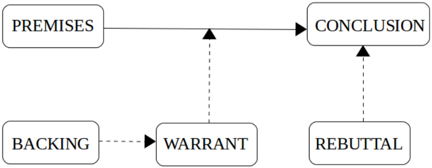

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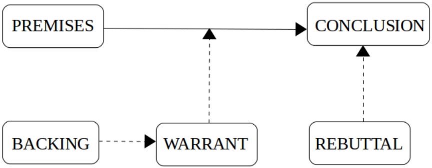

- Argumentation schemes (Walton)

Generally, if *A* then *B* *A* occurs ——————— *B* occurs	CQ1: How strong is the causal generalization? CQ2: Is the evidence cited strong enough to warrant the generalization? CQ3: Are there other factors that would interfere with the effect?

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"The goal of this paper is to give a scientific account of the basic principle "*The one who has the last word laughts best*" of argumentation, and to explore possible ways for implementing this principle on computers"

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$\Rightarrow$ 3-valued acceptability semantics

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- A conflict-free set of arguments S is admissible iff each argument in S is defended by S.

Complete extensions

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Let $(\text{Args}, \text{Att})$ be an AF and $\mathcal{L} : \text{Args} \rightarrow \{\text{in,out,undec}\}$ be a total function. We say that $\mathcal{L}$ is a complete labelling iff:

$\forall A \in \text{Args} : \mathcal{L}(A) = \text{out iff } \exists B \in \text{Args} : ((B, A) \in \text{Att} \wedge \mathcal{L}(B) = \text{in})$, and

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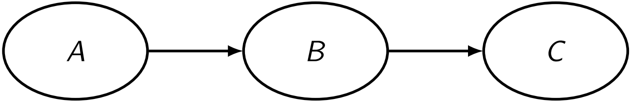

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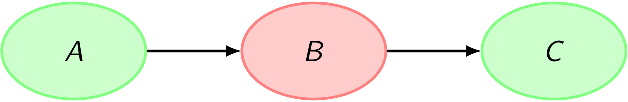

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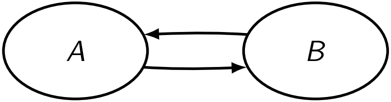

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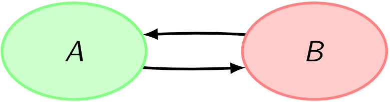

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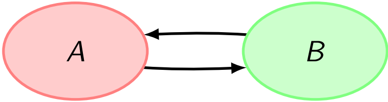

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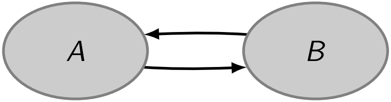

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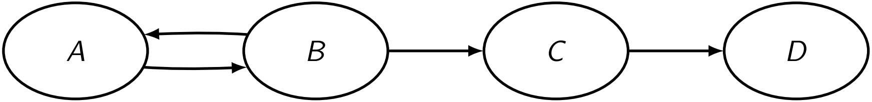

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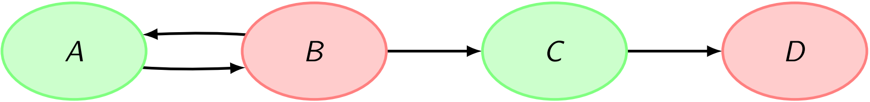

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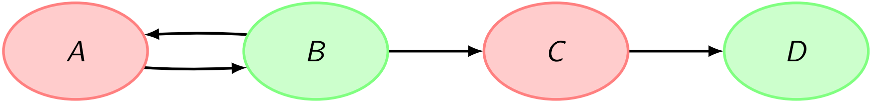

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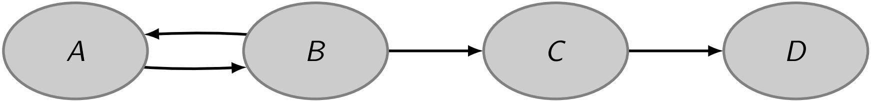

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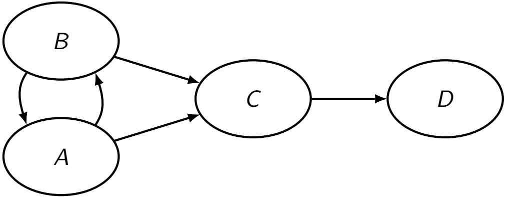

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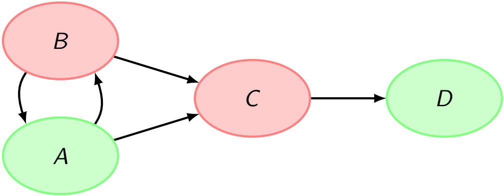

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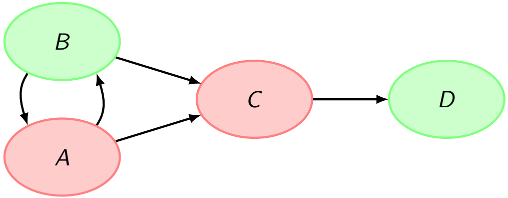

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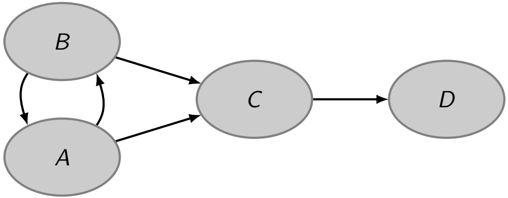

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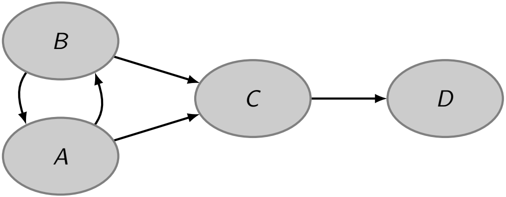

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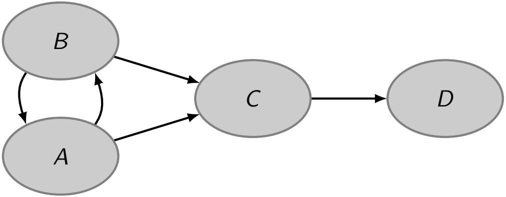

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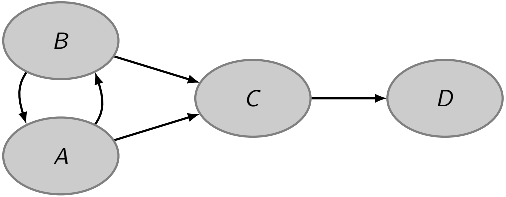

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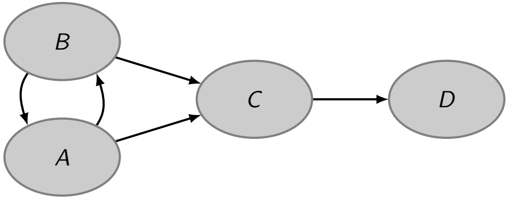

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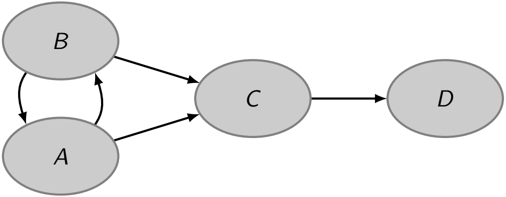

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Skeptical approach: $AF \mid\!\sim A$ iff A is in all preferred extensions

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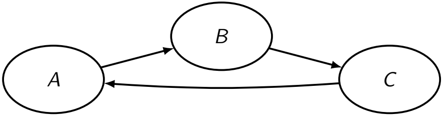

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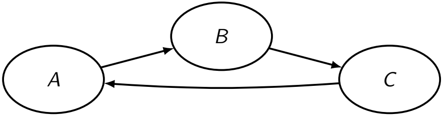

☹ Technically not so well-behaved: the existence of a stable extension is not guaranteed

Semi-stable semantics to the rescue?

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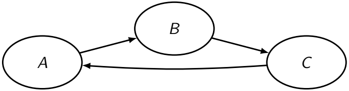

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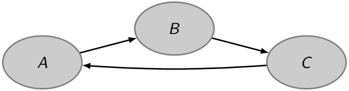

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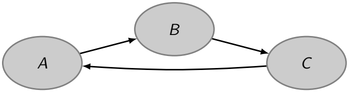

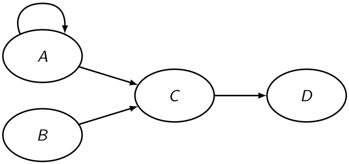

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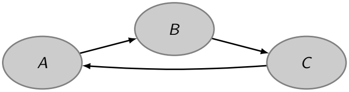

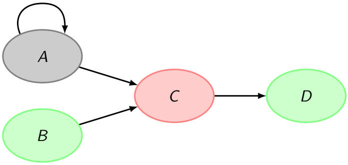

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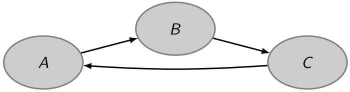

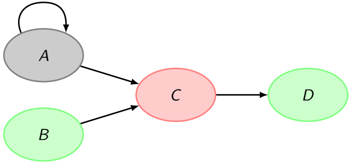

Is $\{B, D\}$ also a stable extension?

So do we always get a semi-stable extension?

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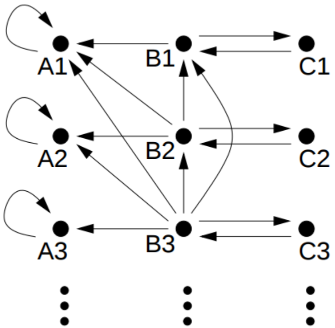
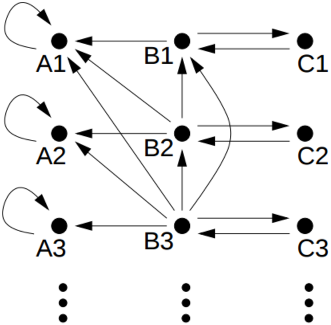


So do we always get a semi-stable extension?

First, note that there are infinitely many preferred extensions:



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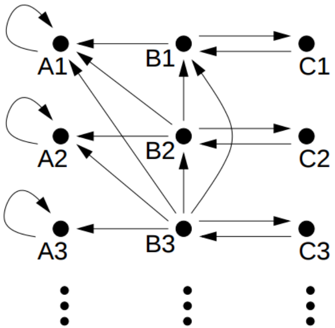

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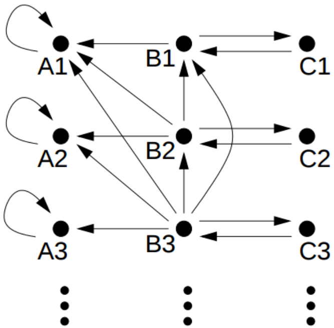

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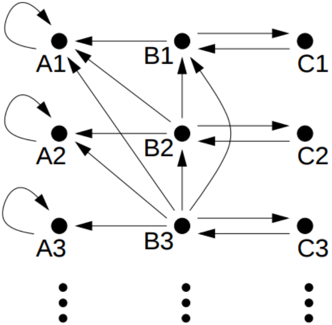

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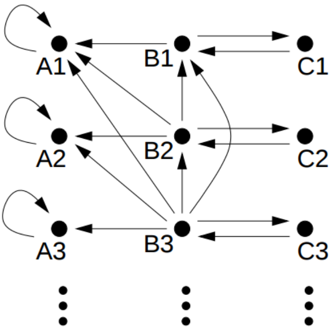
⋮ ⋮



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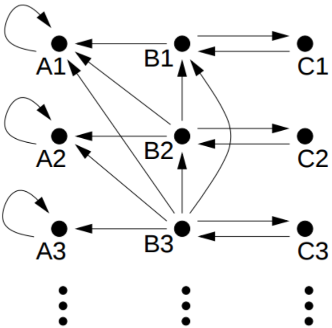

At most one of the B_i can be labelled in. Why?

So do we always get a semi-stable extension? (2)

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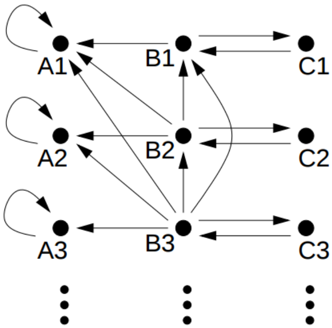

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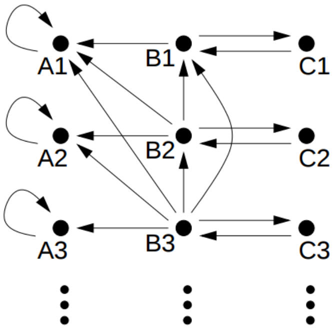

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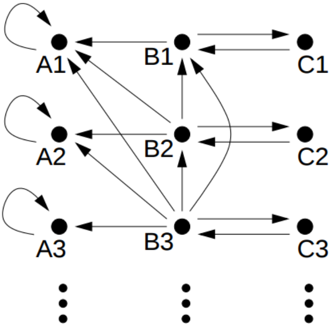

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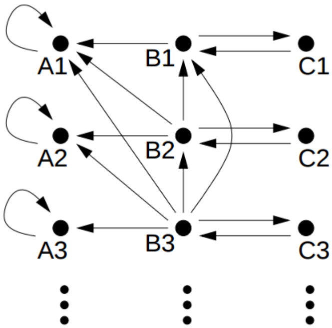

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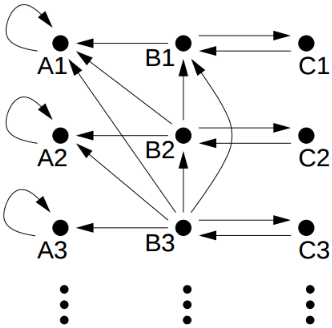

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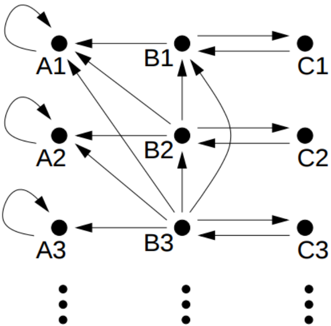

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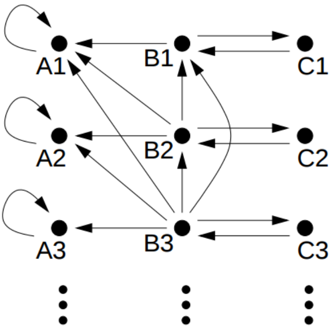

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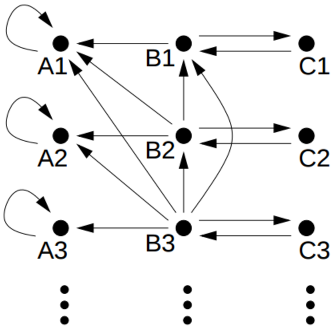

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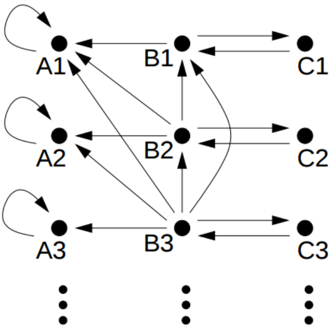

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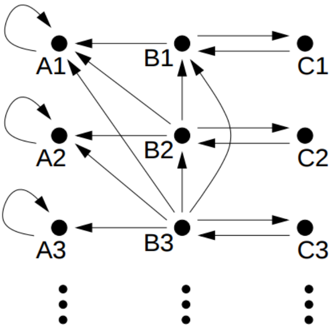

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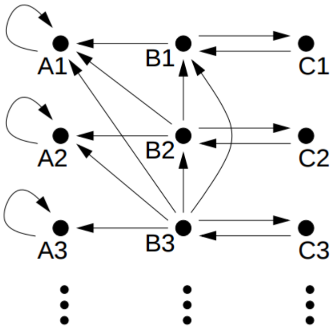

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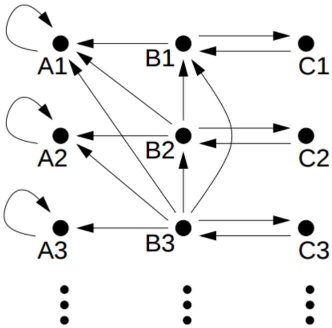

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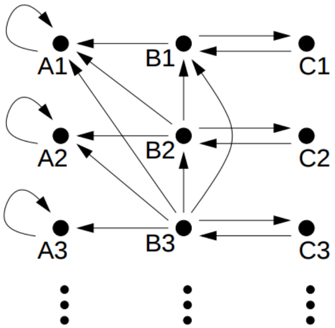

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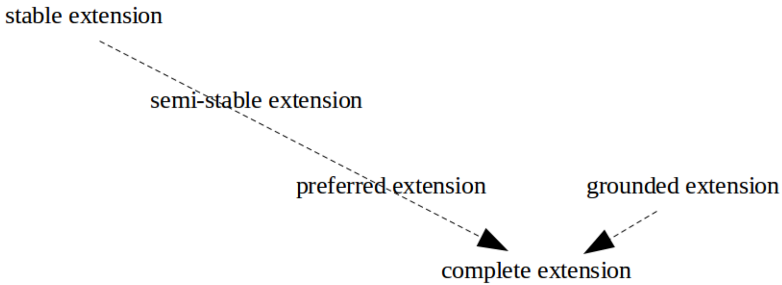
Extensions: overview



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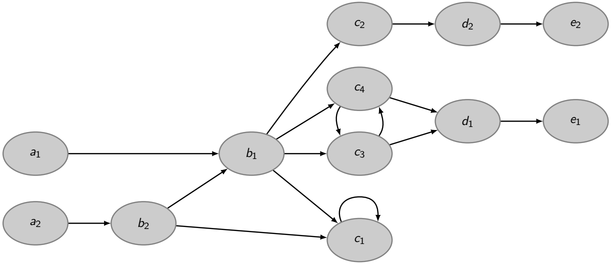

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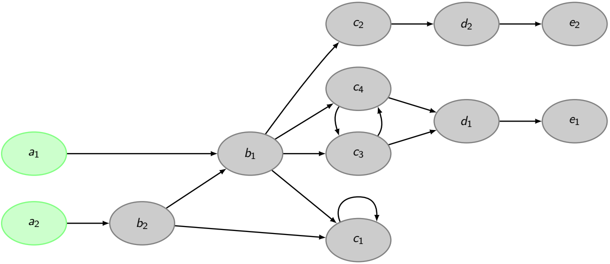

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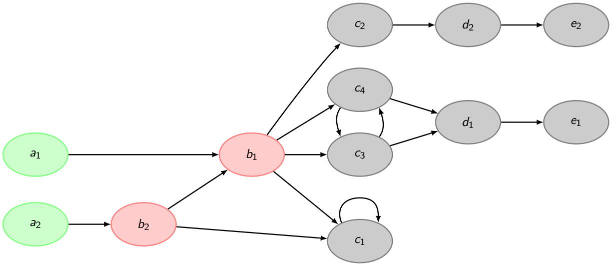

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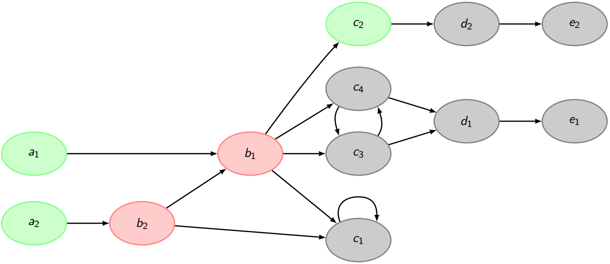

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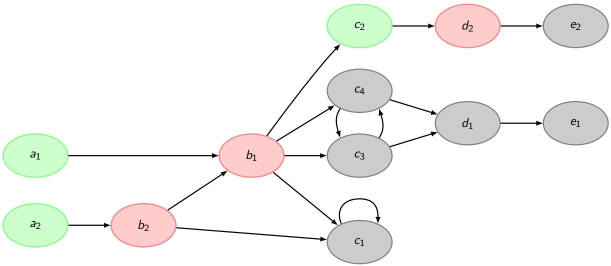

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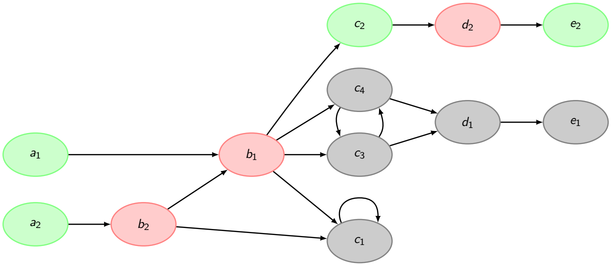

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(ii) More semantics?

- Ideal extension (Dung, Mancarella, Toni): the largest admissible set (w.r.t. set-inclusion) that is a subset of every preferred extension
  $\approx$ unique extension semantics which is less skeptical than the grounded semantics
- Non-admissibility based semantics: obtained extensions need not be admissible
  $\leftrightarrow$ grounded, preferred, (semi-)stable, ideal extensions
- ? Can we call a non-admissible argument *justified*?
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  - Extract an order on arguments (from acceptable to weak)
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(iii) Representing the internal (logical) structure of arguments???