Introduction to Non(-)monotonic Logic

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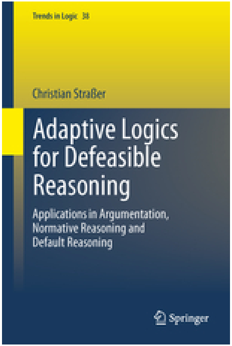
To do list day 2

- Upload slides for days 1 and 2
- Upload bibliography on preferential model semantics (Shoham, KLM) and adaptive logics

1. The adaptive logics framework

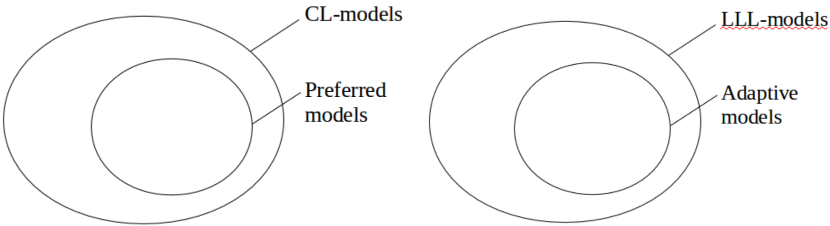


D. Batens. Dynamic dialectical logics. In G. Priest and R. Routley and J. Norman (eds.), *Paraconsistent Logic. Essays on the Inconsistent* (Philosophia Verlag, 1989), pp. 187–217.

D. Batens. A universal logic approach to adaptive logics. *Logica Universalis* 1:221–242 (2007).

C. Straßer. *Adaptive logics for Defeasible Reasoning* (Springer, 2014).

Selection semantics: from Shoham to Batens



- Base: classical logic
- Select $<$-minimal models relative to a given $<$

- Base: lower limit logic (LLL)
- Select least abnormal models relative to
  a set of abnormalities, and
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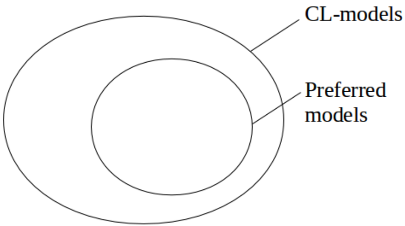

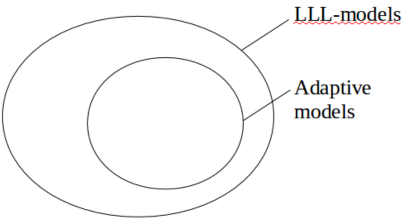

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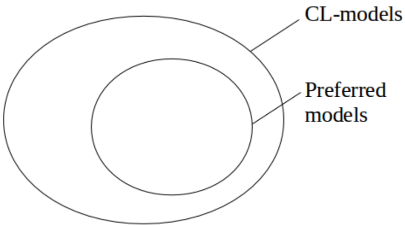

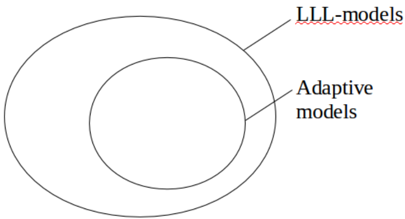

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General idea: to interpret a premise set ‘as normally as possible’ w.r.t. the set of abnormalities.

The standard format

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(i) A lower limit logic **LLL** with the following properties:

- Reflexivity: $\Gamma \subseteq Cn(\Gamma)$
- Transitivity: if $\Delta \subseteq Cn(\Gamma)$ then $Cn(\Delta) \subseteq Cn(\Gamma)$
- Monotony: $Cn(\Gamma) \subseteq Cn(\Gamma \cup \Delta)$
- Compactness: if $\alpha \in Cn(\Gamma)$ then $\alpha \in Cn(\Gamma')$ for some finite $\Gamma' \subseteq \Gamma$

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(ii) A set of abnormalities characterized by a logical form F.

Adaptive logics strengthen their lower limit logic by falsifying abnormalities ‘as much as possible’ relative to the premises.

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(iii) An adaptive strategy

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Extend the assignment function to atomic and negated formulas.

$v(\alpha) = 1$	iff	$v_a(\alpha) = 1$ (where α is an elementary letter)
$v(\sim\alpha) = 1$	iff	$v(\alpha) = 0$ or $v_a(\sim\alpha) = 1$
$v(\alpha \wedge \beta) = 1$	iff	$v(\alpha) = 1$ and $v(\beta) = 1$
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CLuN is paraconsistent: $\alpha \wedge {\sim}\alpha \not\models \beta$

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$\Gamma \models_{\mathbf{CLuN^m}} \alpha$ iff α is verified by all minimally abnormal **CLuN**-models of Γ.

Illustration: the adaptive logic **CLuN^{m}** (2)

$$\Gamma = \{p \wedge r, s \rightarrow \sim q, q \vee \sim p, s \wedge (\sim r \vee q), (p \wedge s) \rightarrow \sim r, \sim p \vee t, q \vee t\}$$

Illustration: the adaptive logic $\mathbf{CLuN^m}$ (2)

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	p	${\sim}p$	q	${\sim}q$	r	${\sim}r$	s	${\sim}s$	t	${\sim}t$	$Ab(M)$
M_1	1	1	1	1	1	1	1	1	1	1	$!p, !q, !r, !s, !t$
M_2	1	1	1	1	1	1	1	1	1	0	$!p, !q, !r, !s$
M_3	1	1	1	1	1	1	1	1	0	1	$!p, !q, !r, !s$
M_4	1	1	1	1	1	1	1	0	1	1	$!p, !q, !r, !t$
M_5	1	1	1	1	1	1	1	0	1	0	$!p, !q, !r$
M_6	1	1	1	1	1	1	1	0	0	1	$!p, !q, !r$
M_7	1	1	0	1	1	1	1	1	1	1	$!p, !r, !s, !t$
M_8	1	1	0	1	1	1	1	1	1	0	$!p, !r, !s$
M_9	1	1	0	1	1	1	1	0	1	1	$!p, !r, !t$
M_{10}	1	1	0	1	1	1	1	0	1	0	$!p, !r$
M_{11}	1	0	1	1	1	1	1	1	1	1	$!q, !r, !s, !t$
M_{12}	1	0	1	1	1	1	1	1	1	0	$!q, !r, !s$
M_{13}	1	0	1	1	1	1	1	0	1	1	$!q, !r, !t$
M_{14}	1	0	1	1	1	1	1	0	1	0	$!q, !r$

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Minimally abnormal models of Γ: M_{10}, M_{14}

$\Gamma \models_{\mathbf{CLuN^m}} t$

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$$\Gamma = \{p \wedge r, s \rightarrow \sim q, q \vee \sim p, s \wedge (\sim r \vee q), (p \wedge s) \rightarrow \sim r, \sim p \vee t, q \vee t\}$$

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Where Δ is a finite subset of Ω, $Dab(\Delta) = \bigvee \Delta$

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Where Δ is a finite subset of Ω, $Dab(\Delta) = \bigvee \Delta$

A minimal *Dab*-consequence of Γ is a formula $Dab(\Delta)$ such that

$\Gamma \models_{\mathbf{CLuN}} Dab(\Delta)$, and

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Where $Dab(\Delta_1), Dab(\Delta_2), \ldots$ are the minimal *Dab* consequences of Γ,
$U(\Gamma) = \{\Delta_1, \Delta_2, \ldots\}$ is the set of formulas that are unreliable w.r.t Γ.

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A **CLuN**-model M of Γ is reliable iff $Ab(M) \subseteq U(\Gamma)$.

Illustration: the adaptive logic **CLuN**$^{\mathbf{r}}$ (2)

$U(\Gamma) = \{p \wedge {\sim}p, q \wedge {\sim}q, r \wedge {\sim}r\}$

	p	$\sim p$	q	$\sim q$	r	$\sim r$	s	$\sim s$	t	$\sim t$	$Ab(M)$
M_1	1	1	1	1	1	1	1	1	1	1	$!p, !q, !r, !s, !t$
M_2	1	1	1	1	1	1	1	1	1	0	$!p, !q, !r, !s$
M_3	1	1	1	1	1	1	1	1	0	1	$!p, !q, !r, !s$
M_4	1	1	1	1	1	1	1	0	1	1	$!p, !q, !r, !t$
M_5	1	1	1	1	1	1	1	0	1	0	$!p, !q, !r$
M_6	1	1	1	1	1	1	1	0	0	1	$!p, !q, !r$
M_7	1	1	0	1	1	1	1	1	1	1	$!p, !r, !s, !t$
M_8	1	1	0	1	1	1	1	1	1	0	$!p, !r, !s$
M_9	1	1	0	1	1	1	1	0	1	1	$!p, !r, !t$
M_{10}	1	1	0	1	1	1	1	0	1	0	$!p, !r$
M_{11}	1	0	1	1	1	1	1	1	1	1	$!q, !r, !s, !t$
M_{12}	1	0	1	1	1	1	1	1	1	0	$!q, !r, !s$
M_{13}	1	0	1	1	1	1	1	0	1	1	$!q, !r, !t$
M_{14}	1	0	1	1	1	1	1	0	1	0	$!q, !r$

Illustration: the adaptive logic **CLuN**$^{\mathbf{r}}$ (2)

$U(\Gamma) = \{p \wedge \sim p, q \wedge \sim q, r \wedge \sim r\}$

	p	$\sim p$	q	$\sim q$	r	$\sim r$	s	$\sim s$	t	$\sim t$	$Ab(M)$
M_1	1	1	1	1	1	1	1	1	1	1	$!p, !q, !r, !s, !t$
M_2	1	1	1	1	1	1	1	1	1	0	$!p, !q, !r, !s$
M_3	1	1	1	1	1	1	1	1	0	1	$!p, !q, !r, !s$
M_4	1	1	1	1	1	1	1	0	1	1	$!p, !q, !r, !t$
M_5	1	1	1	1	1	1	1	0	1	0	$!p, !q, !r$
M_6	1	1	1	1	1	1	1	0	0	1	$!p, !q, !r$
M_7	1	1	0	1	1	1	1	1	1	1	$!p, !r, !s, !t$
M_8	1	1	0	1	1	1	1	1	1	0	$!p, !r, !s$
M_9	1	1	0	1	1	1	1	0	1	1	$!p, !r, !t$
M_{10}	1	1	0	1	1	1	1	0	1	0	$!p, !r$
M_{11}	1	0	1	1	1	1	1	1	1	1	$!q, !r, !s, !t$
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Reliable: M_5, M_6, M_{10}, M_{14}

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M_{10}	1	1	0	1	1	1	1	0	1	0	$!p, !r$
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M_{12}	1	0	1	1	1	1	1	1	1	0	$!q, !r, !s$
M_{13}	1	0	1	1	1	1	1	0	1	1	$!q, !r, !t$
M_{14}	1	0	1	1	1	1	1	0	1	0	$!q, !r$

$\Gamma \models_{\mathbf{CLuN^r}} \alpha$ iff α is verified by all reliable **CLuN**-models of Γ.

Reliable: M_5, M_6, M_{10}, M_{14}

$\Gamma \not\models_{\mathbf{CLuN^r}} t$

Dynamic proofs for adaptive logics

- Defeasibility via *conditional* inferences

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PREM If $\alpha \in \Gamma$:

$$\frac{\vdots \quad \vdots}{\alpha \quad \emptyset}$$

1	$p \wedge r$	PREM	$\emptyset$
2	$q \vee \sim p$	PREM	$\emptyset$
3	$s \rightarrow \sim q$	PREM	$\emptyset$
4	$s \wedge (\sim r \vee q)$	PREM	$\emptyset$
5	$(p \wedge s) \rightarrow \sim r$	PREM	$\emptyset$
6	$\sim p \vee t$	PREM	$\emptyset$
7	$q \vee t$	PREM	$\emptyset$

Dynamic proofs: the unconditional rule

RU If $\alpha_1, \ldots, \alpha_n \vdash_{\mathbf{CLuN}} \beta$:

$$\begin{array}{ll} \alpha_1 & \Delta_1 \\ \vdots & \vdots \\ \alpha_n & \Delta_n \\ \hline \beta & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

1	$p \wedge r$	PREM	$\emptyset$
2	$q \vee {\sim}p$	PREM	$\emptyset$
3	$s \to {\sim}q$	PREM	$\emptyset$
4	$s \wedge ({\sim}r \vee q)$	PREM	$\emptyset$
5	$(p \wedge s) \to {\sim}r$	PREM	$\emptyset$
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6	$\sim p \vee t$	PREM	$\emptyset$
7	$q \vee t$	PREM	$\emptyset$
8	$\sim q$	3,4; RU	$\emptyset$

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5	$(p \wedge s) \rightarrow {\sim}r$	PREM	$\emptyset$
6	${\sim}p \vee t$	PREM	$\emptyset$
7	$q \vee t$	PREM	$\emptyset$
8	${\sim}q$	3,4; RU	$\emptyset$
9	$t \vee (q \wedge {\sim}q)$	7,8; RU	$\emptyset$

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RC If $\alpha_1, \ldots, \alpha_n \vdash_{\textbf{CLuN}} \beta \vee Dab(\Theta)$:

$$\begin{array}{ll} \alpha_1 & \Delta_1 \\ \vdots & \vdots \\ \alpha_n & \Delta_n \\ \hline \beta & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}$$

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2	$q \vee \sim p$	PREM	$\emptyset$
3	$s \rightarrow \sim q$	PREM	$\emptyset$
4	$s \wedge (\sim r \vee q)$	PREM	$\emptyset$
5	$(p \wedge s) \rightarrow \sim r$	PREM	$\emptyset$
6	$\sim p \vee t$	PREM	$\emptyset$
7	$q \vee t$	PREM	$\emptyset$
8	$\sim q$	3,4; RU	$\emptyset$
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6	$\sim p \vee t$	PREM	$\emptyset$
7	$q \vee t$	PREM	$\emptyset$
8	$\sim q$	3,4; RU	$\emptyset$
9	$t \vee (q \wedge \sim q)$	7,8; RU	$\emptyset$
10	t	9; RC	$\{q \wedge \sim q\}$

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7	$q \vee t$	PREM	$\emptyset$
8	$\sim q$	3,4; RU	$\emptyset$
9	$t \vee (q \wedge \sim q)$	7,8; RU	$\emptyset$
10	t	9; RC	$\{q \wedge \sim q\}$
11	t	1,6; RC	$\{p \wedge \sim p\}$

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7	$q \vee t$	PREM	$\emptyset$
8	${\sim}q$	3,4; RU	$\emptyset$
9	$t \vee (q \wedge {\sim}q)$	7,8; RU	$\emptyset$
10	t	9; RC	$\{q \wedge {\sim}q\}$
11	t	1,6; RC	$\{p \wedge {\sim}p\}$
12	${\sim}{\sim}s$	4; RC	$\{s \wedge {\sim}s\}$

Dynamic proofs: marking lines for reliability

Where $Dab(\Delta_1), Dab(\Delta_2), \ldots$ are the minimal *Dab*-formulas derived at stage s on the condition $\emptyset$: $U_s(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$

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Where Δ is the condition of line i derived at stage s, line i is r-marked at s iff $\Delta \cap U_s(\Gamma) \neq \emptyset$.

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6	$\sim p \vee t$	PREM	$\emptyset$
7	$q \vee t$	PREM	$\emptyset$
8	$\sim q$	3,4; RU	$\emptyset$
9	$t \vee (q \wedge \sim q)$	7,8; RU	$\emptyset$
10	t	9; RC	$\{q \wedge \sim q\}$
11	t	1,6; RC	$\{p \wedge \sim p\}$
12	$\sim\sim s$	4; RC	$\{s \wedge \sim s\}$

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6	$\sim p \vee t$	PREM	$\emptyset$
7	$q \vee t$	PREM	$\emptyset$
8	$\sim q$	3,4; RU	$\emptyset$
9	$t \vee (q \wedge \sim q)$	7,8; RU	$\emptyset$
10	t	9; RC	$\{q \wedge \sim q\}$✓
11	t	1,6; RC	$\{p \wedge \sim p\}$✓
12	$\sim\sim s$	4; RC	$\{s \wedge \sim s\}$
13	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1,2,8; RU	$\emptyset$

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13	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1,2,8; RU	$\emptyset$
14	$r \wedge \sim r$	1,4,5; RU	$\emptyset$

Dynamic proofs: final derivability

A formula α is finally derived from Γ at line *i* of a proof at a finite stage *s* iff

- α is the second element of line *i*,
- line *i* is not marked at stage *s*, and
- every extension of the proof in which line *i* is marked, may be further extended such that line *i* is unmarked.

	$\vdots$	$\vdots$	$\vdots$
9	$t \vee (q \wedge \sim q)$	7,8; RU	$\emptyset$
10	t	9; RC	$\{q \wedge \sim q\}\checkmark$
11	t	1,6; RC	$\{p \wedge \sim p\}\checkmark$
12	$\sim\sim s$	4; RC	$\{s \wedge \sim s\}$
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$\Gamma \vdash_{\mathbf{CLuN}^r} \alpha$ iff α is finally derived from Γ at a proof line.

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10	t	9; RC	$\{q \wedge \sim q\}$✓
11	t	1,6; RC	$\{p \wedge \sim p\}$✓
12	$\sim\sim s$	4; RC	$\{s \wedge \sim s\}$
13	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1,2,8; RU	$\emptyset$
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$\Gamma \vdash_{\mathbf{CLuN^r}} \sim\sim s$

$\Gamma \nvdash_{\mathbf{CLuN^r}} t$

	$\vdots$	$\vdots$	$\vdots$
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11	t	1,6; RC	$\{p \wedge \sim p\}$✓
12	$\sim\sim s$	4; RC	$\{s \wedge \sim s\}$
13	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1,2,8; RU	$\emptyset$
14	$r \wedge \sim r$	1,4,5; RU	$\emptyset$

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Theorem (Batens, 2007)

$\Gamma \vdash_{\mathbf{CLuN^r}} \alpha$ *iff* $\Gamma \models_{\mathbf{CLuN^r}} \alpha$.

Dynamic proofs: marking lines for minimal abnormality

Where $Dab(\Delta_1), Dab(\Delta_2), \ldots$ are the minimal Dab-formulas derived from Γ at stage s, $\Phi_s(\Gamma)$ is the set of minimal choice sets of $\{\Delta_1, \Delta_2, \ldots\}$.

	$\vdots$	$\vdots$	$\vdots$
9	$t \vee (q \wedge {\sim}q)$	7,8; RU	$\emptyset$
10	t	9; RC	$\{q \wedge {\sim}q\}$
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12	${\sim}{\sim}s$	4; RC	$\{s \wedge {\sim}s\}$
13	$(p \wedge {\sim}p) \vee (q \wedge {\sim}q)$	1,2,8; RU	$\emptyset$
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$\Phi_{14}(\Gamma) = \{\{p \wedge \sim p, r \wedge \sim r\}, \{q \wedge \sim q, r \wedge \sim r\}\}$

	$\vdots$	$\vdots$	$\vdots$
9	$t \vee (q \wedge \sim q)$	7,8; RU	$\emptyset$
10	t	9; RC	$\{q \wedge \sim q\}$
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12	$\sim\sim s$	4; RC	$\{s \wedge \sim s\}$
13	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1,2,8; RU	$\emptyset$
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Where α is derived at line i of a proof from Γ on condition Δ, line i is *m-marked* at stage s iff

(i) there is no $\Delta' \in \Phi_s(\Gamma)$ s.t. $\Delta' \cap \Delta = \emptyset$, or

(ii) for some $\Delta' \in \Phi_s(\Gamma)$, there is no line at which α is derived on a condition Θ s.t. $\Delta' \cap \Theta = \emptyset$.

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	$\vdots$	$\vdots$	$\vdots$
9	$t \vee (q \wedge \sim q)$	7,8; RU	$\emptyset$
10	t	9; RC	$\{q \wedge \sim q\}$
11	t	1,6; RC	$\{p \wedge \sim p\}$
12	$\sim\sim s$	4; RC	$\{s \wedge \sim s\}$
13	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1,2,8; RU	$\emptyset$
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$\Gamma \vdash_{\mathbf{CLuN^m}} \sim\sim s$

$\Gamma \vdash_{\mathbf{CLuN^m}} t$

	⋮	⋮	⋮
9	$t \vee (q \wedge \sim q)$	7,8; RU	$\emptyset$
10	t	9; RC	$\{q \wedge \sim q\}$
11	t	1,6; RC	$\{p \wedge \sim p\}$
12	$\sim\sim s$	4; RC	$\{s \wedge \sim s\}$
13	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1,2,8; RU	$\emptyset$
14	$r \wedge \sim r$	1,4,5; RU	$\emptyset$

Dynamic proofs: marking lines for minimal abnormality

Where $Dab(\Delta_1), Dab(\Delta_2), \ldots$ are the minimal Dab-formulas derived from Γ at stage s, $\Phi_s(\Gamma)$ is the set of minimal choice sets of $\{\Delta_1, \Delta_2, \ldots\}$.

Where α is derived at line i of a proof from Γ on condition Δ, line i is m-marked at stage s iff

(i) there is no $\Delta' \in \Phi_s(\Gamma)$ s.t. $\Delta' \cap \Delta = \emptyset$, or

(ii) for some $\Delta' \in \Phi_s(\Gamma)$, there is no line at which α is derived on a condition Θ s.t. $\Delta' \cap \Theta = \emptyset$.

$\Phi_{14}(\Gamma) = \{\{p \wedge \sim p, r \wedge \sim r\}, \{q \wedge \sim q, r \wedge \sim r\}\}$

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Theorem (Batens, 2007)

$\Gamma \vdash_{\mathbf{CLuN^m}} \alpha$ *iff* $\Gamma \vDash_{\mathbf{CLuN^m}} \alpha$.

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$$\Gamma \subseteq Cn_{\mathbf{AL}}(\Gamma) \qquad \text{(Reflexivity)}$$

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- If Γ is normal (no *Dab*-formulas are LLL-derivable) then $Cn_{\mathbf{AL^r}}(\Gamma) = Cn_{\mathbf{AL^m}}(\Gamma) = Cn_{\mathbf{ULL}}(\Gamma)$

2. Variations, pitfalls, subtleties

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Take care when defining abnormalities!

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$$\frac{\neg(p \wedge q) \quad \Diamond p \quad \Diamond\Diamond q}{p}$$

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$$\Omega_i^{\Diamond} = \{\Diamond_i \alpha \wedge \neg\alpha\}$$

(iii) Strategy: $\sqsubset$-minimal abnormality (V/D Putte & Straßer, 2012)

The lexicographic order $\sqsubset$: $\Delta \sqsubset \Delta'$ iff

$\langle \Delta \cap \Omega_i \rangle_{i \in I} \sqsubset_{\text{lex}} \langle \Delta' \cap \Omega_i \rangle_{i \in I}$ iff

(1) there is an $i \in I$ such that for all $j < i$, $\Delta \cap \Omega_j = \Delta' \cap \Omega_j$, and

(2) $\Delta \cap \Omega_i \subset \Delta' \cap \Omega_i$.

Combining adaptive logics (2)

The adaptive logic $\mathbf{K^m}$:

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$r \wedge \neg(p \wedge q), \Diamond_1 \neg r, \Diamond_1 p, \Diamond_2 q \vdash_{\mathbf{K^m}} p \wedge \neg q$

A third strategy: normal selections

1	$p \wedge q \wedge r$	PREM	$\emptyset$
2	$(\sim p \vee \sim q) \wedge \sim r$	PREM	$\emptyset$
3	$\sim p \vee s$	PREM	$\emptyset$
4	$\sim q \vee s$	PREM	$\emptyset$
5	$\sim p \vee t$	PREM	$\emptyset$
6	$\sim r \vee u$	PREM	$\emptyset$
7	s	1,3; RC	$\{p \wedge \sim p\}$
8	s	1,4; RC	$\{q \wedge \sim q\}$
9	t	1,5; RC	$\{p \wedge \sim p\}$
10	u	2,6; RC	$\{r \wedge \sim r\}$
11	$(p \wedge \sim p) \vee (q \wedge \sim q)$	1,2; RU	$\emptyset$
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9	t	1,5; RC	$\{p \wedge \sim p\}\checkmark$
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Normal selections: line l with condition Δ is marked at stage s iff $Dab(\Delta)$ is derived at s on the condition $\emptyset$.

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Normal selections: line l with condition Δ is marked at stage s iff $Dab(\Delta)$ is derived at s on the condition $\emptyset$.
A (finite) set $\Delta \subset \Omega$ is *normal* w.r.t. Γ iff $\Gamma \not\models Dab(\Delta)$.
$\Gamma \models_n \alpha$ iff for some $\Delta \subset \Omega$, $\Gamma \models \alpha \vee Dab(Dab)$ while $\Gamma \not\models Dab(\Delta)$.