Introduction to Non(-)monotonic Logic

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ESSLLI 2016, Bolzano
Day 5. Formal argumentation

What is (the structure of) an argument?

- Deductive proofs
- Dialogue games (Lorenz and Lorenzen)
- Toulmin's scheme (1958)
- Argumentation schemes (Walton)

Generally, if A then B occurs

CQ1: How strong is the causal generalization?
CQ2: Is the evidence cited strong enough to warrant the generalization?
CQ3: Are there other factors that would interfere with the effect?
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Generally, if $A$ then $B$

$A$ occurs

$B$ occurs

CQ1: How strong is the causal generalization?

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CQ3: Are there other factors that would interfere with the effect?

- Argumentation schemes (Walton)
Given a collection of arguments, how do we determine which arguments to accept?
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Abstract perspective (Dung, 1995):
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“whether or not a rational agent believes in a statement depends on whether or not the argument supporting this statement can be successfully defended against the counterarguments”

“The goal of this paper is to give a scientific account of the basic principle “The one who has the last word laughts best” of argumentation, and to explore possible ways for implementing this principle on computers”
Dung’s 1995 set-up

An argumentation framework $AF$ is a pair $\langle \text{Args}, \text{Att} \rangle$ where

- $\text{Args}$ is a set of arguments, and
- $\text{Att} \subseteq \text{Args} \times \text{Args}$ is a binary relation of attack between arguments.

We say that argument $A$ attacks argument $B$ iff $(A, B) \in \text{Att}$.

$AF$s are represented as directed graphs. How shall we label nodes in the graph? When are arguments good, acceptable (in)? bad, rejected (out)? unacceptable, undecided (neither)?
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  Good, acceptable (in)?
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⇒ 3-valued acceptability semantics
A set $S$ of arguments is conflict-free if there are no arguments $A$ and $B$ in $S$ such that $A$ attacks $B$. 
Desiderata for acceptable (sets of) arguments

- A set $S$ of arguments is conflict-free if there are no arguments $A$ and $B$ in $S$ such that $A$ attacks $B$.
- Argument $A \in \text{Args}$ is acceptable w.r.t. a set $S$ of arguments iff for each $B \in \text{Args}$: if $B$ attacks $A$ then $B$ is attacked by $S$. ($S$ defends $A$.)
Desiderata for acceptable (sets of) arguments

- A set $S$ of arguments is conflict-free if there are no arguments $A$ and $B$ in $S$ such that $A$ attacks $B$.
- Argument $A \in \text{Args}$ is acceptable w.r.t. a set $S$ of arguments iff for each $B \in \text{Args}$: if $B$ attacks $A$ then $B$ is attacked by $S$. ($S$ defends $A$.)
- A conflict-free set of arguments $S$ is admissible iff each argument in $S$ is defended by $S$. 
Complete extensions

An admissible set $S$ of arguments is a complete extension iff each argument, which is acceptable w.r.t. $S$, belongs to $S$. 
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Complete extensions correspond to complete labellings in the following way:

Let $(\text{Args}, \text{Att})$ be an AF and $\mathcal{L} : \text{Args} \rightarrow \{\text{in, out, undec}\}$ be a total function. We say that $\mathcal{L}$ is a complete labelling iff:

\[
\forall A \in \text{Args} : \mathcal{L}(A) = \text{out} \text{ iff } \exists B \in \text{Args} : ((B, A) \in \text{Att} \land \mathcal{L}(B) = \text{in}), \text{ and}
\]

\[
\forall A \in \text{Args} : \mathcal{L}(A) = \text{in} \text{ iff } \forall B \in \text{Args} : ((B, A) \in \text{Att} \supset \mathcal{L}(B) = \text{out}).
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$\forall A \in \text{Args} : \mathcal{L}(A) = \text{in} \iff \forall B \in \text{Args} : ((B, A) \in \text{Att} \supset \mathcal{L}(B) = \text{out})$. 

\[
\begin{align*}
A & \rightarrow B \\
B & \rightarrow C
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and

$$\forall A \in \text{Args} : \mathcal{L}(A) = \text{in} \iff \forall B \in \text{Args} : ((B, A) \in \text{Att} \supset \mathcal{L}(B) = \text{out}).$$

Complete extension: $\{A, C\}$
What are the complete extensions?

A $\xrightarrow{\text{}}$ B

Complete extensions: \{A\}, \{B\}, \emptyset
What are the complete extensions?

Complete extensions: \{A\}
What are the complete extensions?

Complete extensions: \{A\}, \{B\}
What are the complete extensions?

Complete extensions: \( \{A\}, \{B\}, \emptyset \)
What are the complete extensions?

Complete extensions:

- \{A, C\}
- \{B, D\}
- ∅
What are the complete extensions?

Complete extensions: \{A, C\}
What are the complete extensions?

Complete extensions: \(\{A, C\}, \{B, D\}\)
What are the complete extensions?

Complete extensions: \( \{A, C\}, \{B, D\}, \emptyset \)
Preferred and grounded extensions

A preferred extension of an argumentation framework $AF$ is a maximal (w.r.t. set inclusion) admissible set of $AF$. From the labelling perspective, preferred extensions coincide with those labellings in which in is maximal and out is maximal (and undec is minimal).

The grounded extension of an argumentation framework $AF$ is a minimal (w.r.t. set inclusion) admissible set of $AF$. From the labelling perspective, the grounded extension coincides with those labellings in which in is minimal and out is minimal (and undec is maximal).
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Floating conclusions again

What are the complete extensions?

$$\{A, D\}, \{B, D\}, \emptyset$$

Preferred extensions?

$$\{A, D\}, \{B, D\}$$

Grounded extension?

$$\emptyset$$

Credulous approach:

$$\text{AF} | \neg A \iff A \text{ is in some preferred extension}$$

Skeptical approach:

$$\text{AF} | \neg A \iff A \text{ is in all preferred extensions}$$
Floating conclusions again

What are the complete extensions?

\{A, D\}
Floating conclusions again

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Credulous approach: $AF \models A$ iff $A$ is in some preferred extension
Floating conclusions again

What are the complete extensions?

\{A, D\}, \{B, D\}, \emptyset

Preferred extensions? \{A, D\}, \{B, D\}

Grounded extension? \emptyset

Credulous approach: \(AF \models A\) iff \(A\) is in some preferred extension

Skeptical approach: \(AF \models A\) iff \(A\) is in all preferred extensions
A conflict-free set of arguments $S$ is a **stable extension** iff $S$ attacks each argument which does not belong to $S$. 

What are the stable extensions of the following AF?

Technically not so well-behaved: the existence of a stable extension is not guaranteed.
Stable semantics

A conflict-free set of arguments $S$ is a stable extension iff $S$ attacks each argument which does not belong to $S$.

≈ Good position to be in (informally)
A conflict-free set of arguments \( S \) is a stable extension iff \( S \) attacks each argument which does not belong to \( S \).

≈ Good position to be in (informally)

▶ No arguments are labelled undec
Stable semantics

A conflict-free set of arguments $S$ is a stable extension iff $S$ attacks each argument which does not belong to $S$.

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What are the stable extensions of the following $AF$?

![Diagram of arguments A, B, and C with arrows indicating relationships.]
Stable semantics

A conflict-free set of arguments $S$ is a stable extension iff $S$ attacks each argument which does not belong to $S$.

≈ Good position to be in (informally)

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What are the stable extensions of the following $AF$?

Technically not so well-behaved: the existence of a stable extension is not guaranteed
Given an AF $\langle\text{Args,Att}\rangle$ with $S \subseteq \text{Args}$, let
$S^+ = \{B \mid (A, B) \in \text{Att} \text{ for some } A \in S\}$
Semi-stable semantics to the rescue?

Given an AF \( \langle \text{Args,Att} \rangle \) with \( S \subseteq \text{Args} \), let

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S^+ = \{ B \mid (A, B) \in \text{Att} \text{ for some } A \in S \}
\]

Let \( \langle \text{Args,Att} \rangle \) be an AF, and \( S \subseteq \text{Args} \). \( S \) is a **semi-stable extension** iff \( S \) is a complete extension and \( S \cup S^+ \) is maximal.
Semi-stable semantics to the rescue?

Given an AF $\langle \text{Args}, \text{Att} \rangle$ with $S \subseteq \text{Args}$, let $S^+ = \{ B \mid (A, B) \in \text{Att} \text{ for some } A \in S \}$

Let $\langle \text{Args}, \text{Att} \rangle$ be an AF, and $S \subseteq \text{Args}$. $S$ is a semi-stable extension iff $S$ is a complete extension and $S \cup S^+$ is maximal.

▷ In semi-stable semantics the arguments labelled undec are minimized
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- In semi-stable semantics the arguments labelled undec are minimized
- What are the semi-stable extensions of the following AFs?
Given an AF $\langle \text{Args}, \text{Att} \rangle$ with $S \subseteq \text{Args}$, let

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\[ \begin{array}{ccc}
A & \leftrightarrow & B \\
& \leftrightarrow & C
\end{array} \]
Semi-stable semantics to the rescue?

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![Diagram of the two AFs]
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- In semi-stable semantics the arguments labelled undec are minimized
- What are the semi-stable extensions of the following AFs?

[Diagram of AFs: AF1 with A -> B -> C, AF2 with A -> C, C -> D]
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▶ In semi-stable semantics the arguments labelled undec are minimized

▶ What are the semi-stable extensions of the following AFs?

![Diagram of AFs]

Is \( \{B, D\} \) also a stable extension?
So do we always get a semi-stable extension?

First, note that there are infinitely many preferred extensions:

1. Each $C_i$ is in, each $B_i$ is out, and each $A_i$ is undec.
2. $C_1$ is out, all other $C_i$ are in; $B_1$ is in, all other $B_i$ are out; $A_1$ is out, all $A_j$ with $j > 1$ are undec.
3. $C_2$ is out, all other $C_i$ are in; $B_2$ is in, all other $B_i$ are out; $A_1$ and $A_2$ are out, all $A_j$ with $j > 2$ are undec.

... At most one of the $B_i$ can be labelled in. Why?
So do we always get a semi-stable extension?

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At most one of the $B_i$ can be labelled in. Why?
So do we always get a semi-stable extension? (2)

(Remember: we want to minimize undec)

The larger the $i$ in $B_i$, the less arguments are undec. Ad infinitum.

There is no semi-stable extension.

Each finite AF has at least one semi-stable extension (Caminada, Verheij).
So do we always get a semi-stable extension? (2)

(Remember: we want to minimize undec)
Assume $B_i$ is in. Then $C_i$ is out.

\[
\begin{array}{c}
A_1 \rightarrow B_1 \rightarrow C_1 \\
A_2 \rightarrow B_2 \rightarrow C_2 \\
A_3 \rightarrow B_3 \rightarrow C_3 \\
\vdots & \vdots & \vdots
\end{array}
\]

▶ The larger the $i$ in $B_i$, the less arguments are undec. Ad infinitum.

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So do we always get a semi-stable extension? (2)

(Remember: we want to minimize undec)

Assume $B_i$ is in. Then $C_i$ is out.

For all $j \neq i$: $B_j$ is out and $C_j$ is in.

▶

The larger the $i$ in $B_i$, the less arguments are undec. Ad infinitum.

/ There is no semi-stable extension.

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So do we always get a semi-stable extension? (2)

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Assume $B_i$ is in. Then $C_i$ is out.

For all $j \neq i$: $B_j$ is out and $C_j$ is in.

All $A_k$ with $k \leq i$ are out.

All $A_k$ with $k > i$ are undec.

▶

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So do we always get a semi-stable extension? (2)

(Remember: we want to minimize undec)
Assume $B_i$ is in. Then $C_i$ is out.
For all $j \neq i$: $B_j$ is out and $C_j$ is in.
All $A_k$ with $k \leq i$ are out.
All $A_k$ with $k > i$ are undec.

For instance:
If $B_1$ is in, then:
- $B_2, B_3, \ldots$ are out

The larger the $i$ in $B_i$, the less arguments are undec. Ad infinitum.

There is no semi-stable extension.
Each finite AF has at least one semi-stable extension (Caminada, Verheij).
So do we always get a semi-stable extension? (2)

(Remember: we want to minimize undec)
Assume \( B_i \) is in. Then \( C_i \) is out.
For all \( j \neq i \): \( B_j \) is out and \( C_j \) is in.
All \( A_k \) with \( k \leq i \) are out.
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For instance:
If \( B_1 \) is in, then:
- \( B_2, B_3, \ldots \) are out
- \( C_1 \) is out; \( C_2, C_3, \ldots \) are in
So do we always get a semi-stable extension? (2)

(Remember: we want to minimize undec)

Assume $B_i$ is in. Then $C_i$ is out.
For all $j \neq i$: $B_j$ is out and $C_j$ is in.
All $A_k$ with $k \leq i$ are out.
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For instance:
If $B_1$ is in, then:
- $B_2, B_3, \ldots$ are out
- $C_1$ is out; $C_2, C_3, \ldots$ are in
- $A_1$ is out; $A_2, A_3, \ldots$ are undec
So do we always get a semi-stable extension? (2)

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So do we always get a semi-stable extension? (2)

(Remember: we want to minimize undec)

Assume $B_i$ is in. Then $C_i$ is out.
   For all $j \neq i$: $B_j$ is out and $C_j$ is in.
All $A_k$ with $k \leq i$ are out.
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Extension properties

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An argumentation framework is well-founded iff there exists no infinite sequence $A_0, A_1, A_2, \ldots$ such that for each $A_i$, $A_{i+1}$ attacks $A_i$.

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Extensions: overview

stable extension

semi-stable extension

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grounded extension

complete extension
Construction of the grounded extension

The grounded extension $G$ relative to an AF $(A, \text{Att})$ is defined as follows (where $A$ is countable):

(i) $G_0$: the set of all arguments in $A$ without attackers;
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(ii) More semantics?
   ▶ Ideal extension (Dung, Mancarella, Toni): the largest admissible set (w.r.t. set-inclusion) that is a subset of every preferred extension ≈ unique extension semantics which is less skeptical than the grounded semantics
   ▶ Non-admissibility based semantics: obtained extensions need not be admissible ↔ grounded, preferred, (semi-)stable, ideal extensions
   ▶ Can we call a non-admissible argument justified?
   ▶ Ranking-based semantics (Amgoud, Ben-Naim)
     - Extract an order on arguments (from acceptable to weak)
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